Reg. No. :

Question Paper Code : 95388

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Third Semester

Computer Technology

XCS 231/10677 SW 301— PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS

(Common to 5 Year M.Sc. Software Engineering and 5 Year M.Sc. Information Technology)

(Regulations 2003/2007/2010)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the general solution of $pyz + qzx = xy$.

2. Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 0$.

3. Define Fourier series of $f(x)$ in $(c, c+2l)$.

4. State Parseval's theorem, in Fourier series.

5. State the Fourier integral representation of $f(x)$.

6. If the Fourier transform of $f(x)$ is $\overline{f(s)}$, find $\frac{d^r}{ds^r}\bar{f}(s)$.

7. Find $L\left(e^{-2t}\cos t\right)$.

8. Verify final value theorem of Laplace transform for $f(t) = 1 - e^{-at}$.

9. If $z[f(n)] = F(z)$ then prove that $z\left[a^{-n}f(n)\right] = F(az)$.

10. Find the z-transform of $\left[\sin\frac{n\pi}{2}\right]$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Solve the equation : $(pq - p - q)(z - px - qy) = pq$. (8)

(ii) Solve $2y(z-3)p + (2x - z)q = y(2x - 3)$. (8)

Or

(b) (i) Form the differential equation by eliminating f and g from $Z = f(y) + g(x + y + z)$. (7)

(ii) Solve the equation $(D^2 - 2DD' + D'^2)z = x^2y^2e^{x+y}$. (9)

12. (a) (i) Find the half-range sine series of $f(x) = a$ in $(0, l)$. Deduce the sum of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...\infty$.

(ii) Find the half-range cosine series of $f(x) = x$ in (0, 1). Deduce the sum of the series $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ...\infty$.

Or

(b) Find the Fourier series expansion of $f(x) = x^2 + x$ in $(-2, 2)$. Hence find the sum of the series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...\infty$.

13. (a) (i) Obtain the Fourier expansion of $f(x) = x(2\pi - x)$ in $0 < x < 2\pi$.

(ii) Find the Fourier series of $f(x) = \begin{cases} x + 1, 0 < x < \pi \\ x - 1, -\pi < x < 0 \end{cases}$

Or

(b) (i) Obtain the Fourier sine series of unity in $0 < x < \pi$. Hence show that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + ...$

(ii) Find the Fourier cosine series of $f(x) = (lx - x^2)$ in $(0, l)$.

14. (a) (i) Using convolution theorem find the inverse Laplace transform of $\left(\frac{s}{(s^2 + a^2)^2}\right)$.

(ii) Find the Laplace transform of $f(t) = \begin{cases} t & for\ 0 < t < a \\ 2a - t & for\ a < t < 2a \end{cases}$ where $f(t + 2a) = f(t)$.

Or

(b) (i) Using Laplace transform, solve for y the integral equation $y(t) = t^2 + \int_0^t y(u)\sin(t-u)du$.

(ii) Find the inverse Laplace transform of $\left[\log\left(\frac{s+a}{s+b}\right)\right]$.

15. (a) (i) Find $Z(r^n \cos n\theta)$, $Z(r^n \sin \theta)$, $Z(\cos n\theta)$ and $Z(\sin n\theta)$. (10)

(ii) Find the Z-transform of $f(n) = \frac{2n+3}{(n+1)(n+2)}$. (6)

Or

(b) (i) Solve the equation $f(n) + 3f(n-1) - 4f(n-2) = 0, n \geq 2$, given that $f(0) = 3$ and $f(1) = -2$. (10)

(ii) Find the inverse Z-transform of $\frac{z}{z^2+7z+10}$. (6)
