Reg. No. :

Question Paper Code : 95393

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fourth Semester

Computer Technology

XCS 241/10677 SW 401 — DISCRETE MATHEMATICS

(Common to 5 Year M.Sc. Software Engineering/5 Year M.Sc. Information Technology)

(Regulations 2003/2007/2010)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define conditional statements. Write the truth table.

2. Define equivalence of formulas.

3. Give an example of a relation which is both symmetric and antisymmetric.

4. What is meant by an equivalence relation? Explain with an example.

5. Show that in a group $(G, *)$ the identity element is unique.

6. If $f: G_1 \to G_2$ is a group homomorphism, show that Kerf is a normal subgroup of G_1.

7. Prove that $a \cdot 0 = 0 = 0 \cdot a$ in a ring R.

8. Define a polynomial ring.

9. What is power set? Give an example.

10. State Demorgan's laws on a complemented distributive lattice $(L, \wedge, \vee)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain PDNF and PCNF for $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$. (8)

(ii) Determine validity of the following argument. My father praises me only if I can be proud of myself. Either I do well in sports or I cannot be proud of myself. If I study hard, then I cannot do well in sports. Therefore, if father praises me, then I do not study well. (8)

Or

(b) (i) Verify whether $(q \vee r) \rightarrow (p \wedge \sim r)$ is a tautology. (8)

(ii) Show that $n^3 + 2n$ is divisible by 3 for all $n \geq 1$ by method of induction? (8)

12. (a) (i) Let $X = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{\langle x, y\rangle / x - y \text{ is divisible by } 3\}$. Prove that R is an equivalence relation. Draw the graph of R. (8)

(ii) Let $A = \{1, 2, 3\}$. Define $f : A \rightarrow A$ by $f(1) = 2$, $f(2) = 1$ and $f(3) = 3$. Find f^2, f^4, f^{-1}. (8)

Or

(b) (i) Let R denote a relation on the set of all ordered pairs of positive integers such that $\langle x, y\rangle R \langle u, v\rangle$ if and only if $xv = yu$. Show that R is an equivalence relation. (8)

(ii) If $f : A \rightarrow B$ and $g : B \rightarrow C$ are onto functions. Prove that the product function $(g \circ f) : A \rightarrow C$ is also onto. (8)

13. (a) (i) Show that in a group $< G, * >$, if for any $a, b \in G$, $(a * b)^2 = a^2 * b^2$, then $< G, * >$ must be abelian. (8)

(ii) Show that the set of all elements a of a group $< G, * >$ such that $a * x = x * a$ for every $x \in G$ is a subgroup of G. (8)

Or

(b) (i) Show that among the cosets determined by a subgroup S in a group $< G, * >$, only one of the cosets is a subgroup. (8)

(ii) Show that every subgroup of a cyclic group is normal. (8)

14. (a) (i) State and prove any four properties of a ring. (8)

(ii) Prove that every field is an integral domain. (8)

Or

(b) (i) If a ring R is an integral domain, prove that the polynomial ring $R[x]$ is also an integral domain. (10)

(ii) If R is an integral domain with unity element, prove that every unit in $R[x]$ is a unit in R. (6)

15. (a) (i) In a lattice $(L, \leq, \wedge, \vee)$, prove that the following are equivalent. For $a, b \in L$,

(1) $a \leq b$

(2) $a \vee b = b$

(3) $a \wedge b = a$.

(ii) Show that in a lattice $(L, \leq, \wedge, \vee)$, for $a, bc \in L$, $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$.

Or

(b) (i) Prove that every chain is a distributive Lattice.

(ii) In a Boolean Algebra, prove that $(a \wedge b)' = a' \vee b'$ and $(a \vee b)' = a' \wedge b'$, for all $a, b \in L$.