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Question Paper Code : 95378

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

First Semester

Computer Technology

XCS 112/ 10677 SW 102 — TRIGONOMETRY, ALGEBRA AND CALCULUS

(Common to M.Sc. Information Technology and M.Sc. Software Engineering)

(Regulations 2003/2007/2010)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $x+\frac{1}{x}=2\cos\theta$, find $x^n+\frac{1}{x^n}$.

2. Prove that $i^i = e^{-(4n+1)\frac{\pi}{2}}$.

3. Find the rank of the matrix $A=\begin{bmatrix} 3 & 4 & -6 \\ 2 & -1 & 7 \\ 1 & -2 & 8 \end{bmatrix}$.

4. Find the sum of the eigenvalues of 2*A*, if $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

5. If $u=\sin^{-1}(x-y)$, where $x=3t$, $y=4t^3$, find $\frac{du}{dt}$.

6. If $u=2xy$, $\upsilon=x^2-y^2$, $x=r\cos\theta$, $y=r\sin\theta$, compute $\frac{\partial(u,v)}{\partial(r,\theta)}$.

7. Show that $\int_0^{\pi} x\, f(\sin x)dx=\frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$.

8. Evaluate $\int_0^{\pi/2} \sin^9 \theta \, d\theta$.

9. Find the particular Integral for $(D^2 + a^2)y = \sin ax$.

10. Give the general form of Legendre's linear equation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that (8)

$$(1+\cos\theta+i.\sin\theta)^n+(1+\cos\theta-i.\sin\theta)^n = 2^{n+1}.\cos^n\left(\frac{\theta}{2}\right).\cos\left(\frac{n\theta}{2}\right).$$

(ii) If $\sin\theta = \tan hx$, prove that $\tan\theta = \sin hx$. (8)

Or

(b) (i) Express $\frac{\sin 7\theta}{\sin\theta}$ interms of power of $\sin\theta$. (8)

(ii) Separate into the real and imaginary parts of $\tan^{-1}(x+iy)$. (8)

12. (a) (i) If α and β are the roots of $x^2-2x+4=0$, prove that $\alpha^n+\beta^n = 2^{n+1}.\cos\left(\frac{n\pi}{3}\right)$. (8)

(ii) Separate into the real and imaginary parts of $\tan h(x+iy)$. (8)

Or

(b) (i) Expand $\sin^4\theta.\cos^3\theta$ in a series of cosines of multiples of θ. (8)

(ii) If $\cosh u = \sec\theta$, show that $u = \log\left[\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]$. (8)

13. (a) (i) Use Tailor's series theorem to expand $f(x,y) = x^3+xy^2+y^3$ in powers of $(x-1)$ and $(y-2)$. (8)

(ii) Identify the saddle point and the extremum points of $f(x,y) = x^4-y^4-2x^2+2y^2$. (8)

Or

(b) Show that if the perimeter of a triangle is a constant, its area is maximum when it is equilateral. (16)

14. (a) (i) Obtain the reduction formula for $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$ with different cases for in or n odd and even numbers. (8)

(ii) Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = x$ by double integration. (8)

Or

(b) (i) Evaluate $\iiint_V xyz\, dz\, dy\, dx$, where V is the region inside the tetrahedran bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (8)

(ii) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 3$ and $z = 0$. (8)

15. (a) (i) Solve $(D^2 + D + 1)y = e^x \sin^2\left(\frac{x}{2}\right)$. (8)

(ii) Solve $(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$. (8)

Or

(b) (i) Solve the system of equations (8)

$$\frac{dx}{dt} - y = t$$

$$\frac{dy}{dt} + x = t^2$$

(ii) Find the solution of the differential equation $(x^2D^2 + xD + 1)y = \log x \sin(\log x)$. (8)