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# Question Paper Code : 23531

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fourth Semester

Information Technology

MA 1253/MA 1259 — PROBABILITY AND STATISTICS

(Common to Production Engineering, Automobile Engineering, Mechanical Engineering, Textile Technology, Textile Technology (Textile Chemistry) and Textile Technology (Fashion Technology)

(Also common to Sixth Semester – Civil Engineering)

(Regulation 2004/2007)

(Common to B.E. (Part-Time) Third Semester, Mechanical Engineering – Regulation 2005)

Time : Three hours　　　　Maximum : 100 marks

(Use of approved statistical table is permitted)

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. A committee consists of 9 students two of which are from $1^{st}$ year, three from $2^{nd}$ years and four from $3^{rd}$ year. Three students are to be removed at random. What is the chance that two belong to the same class and third to the different class?

2. Is the function defined as follows a density function?

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

3. Comment on the validity of the following statement :

   'X is a binomial random variable with mean 3 and variance 4'.

4. "Attempts are made until success is seen. Number of attempts made is not given any priority to see the success." Mention the name of the random variable and the underlying property.

5. Suppose that $p(x, y)$, the joint probability mass function of $X$ and $Y$ is given by $p(0, 0) = 0.4$, $p(0, 1) = 0.2$, $p(1, 0) = 0.1$, $p(1, 1) = 0.3$. Calculate the conditional probability mass function of $X$, given that $Y = 1$.

6. State Central Limit Theorem.

7. Mention any two applications of Chi-square distribution.

8. Define level of significance.

9. State the basic assumption in the analysis of variance.

10. Define Latin square design.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A company has three factories $F_1$, $F_2$ and $F_3$ producing respectively 35%, 15% and 50% of the total output. The probabilities for producing a non-defective item by the three factories are respectively 0.75, 0.95 and 0.85. All the items produced are put in one stock pile. One item is chosen at random and it is found to be defective What is the probability that it came from factory $F_2$? (8)

(ii) Obtain the moment generating function of the random variable X whose pdf given by $f(x) = \frac{1}{c} e^{-x/c}$, $0 \le x < \infty$, $c > 0$. Hence, find the mean and variance of $X$. (8)

Or

(b) (i) A discrete random variable $X$ has the following probability distribution.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $X = x$: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $P\{X = x\}$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2 + k$ |

Find the value of $k$.

What is the smallest value of $\lambda$ for which $P\{X \le \lambda\} > 1/2$

Show that $P\{1.5 < X < 4.5 | X > 2\} = 5/7$. (8)

(ii) A box contains 6 red, 4 white and 5 blue balls. Four balls are drawn at random, Find the probability that among the balls drawn, there is at least one ball of each colour. (8)

12. (a) (i) The mean and standard deviation of marks in Mathematics are '70 and 10 respectively. the corresponding values for control systems are 55 and 15 respectively. Assume that the marks in the two subjects are independent normal variates, find the probability that a student scores a total of marks lying between 100 and 120 in the two subjects? (10)

(ii) Derive the mean and variance of Gamma distribution. (6)

Or

(b) (i) Find the mean and variance of Weibull distribution (10)

(ii) In a certain industrial facility accidents occur in frequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other. What is the probability that is any given period of 400 days there will be an accident on one day? (6)

13. (a) (i) If two random variables have the joint probability density

$$f(x_1, x_2) = \begin{cases} 2/3(x_1 + 2x_2) & \text{for } 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases},$$ find the conditional density of first given that the second, takes on the value $x_2$. (6)

(ii) If $X$ and $Y$ are two random variables having joint density function

$$f(x,y) = \begin{cases} \frac{1}{8}(6 - x - y); & 0 < x < 2,\ 2 < y < 4 \\ 0; & \text{otherwise} \end{cases}.$$ Find $P(X < 1 \cap Y < 3)$ and $P(X < 1 / Y < 3)$ (10)

Or

(b) (i) Let $(X, Y)$ be the two dimensional random variable described by the joint p.d.f. $f(x,y) = \begin{cases} 8xy, & 0 \le x \le 1,\ 0 \le y \le x \\ 0, & \text{elsewhere} \end{cases}$ find the $Cov(x, y)$. (8)

(ii) The joint p.d.f. of two random variables $X$ and $Y$ is given by

$$f(x, y) = \begin{cases} k\{(x + y) - (x^2 + y^2)\}, & 0 < (x, y) < 1 \\ 0, & \text{otherwise} \end{cases}$$

Prove that $X$ and $Y$ are uncorrelated but not independent. (8)

14. (a) (i) In a large city A, 20% of a random sample of 900 school boys were consumers of tea. In another large city, 185% of random sample of 1600 school boys were consumers of tea. Is the difference between the proportions significant? (8)

(ii) Tests made on the breaking strength of ten pieces of a metal gave the following results : 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 kg. Test if the mean breaking strength of the wire can be assumed as 577 kg. (8)

Or

(b) (i) The nicotine contents in two random samples of tobacco are as follows. (8)

| | | | | | | |
|---|---|---|---|---|---|---|
| Sample I : | 21 | 24 | 25 | 26 | 27 | |
| Sample II : | 22 | 27 | 28 | 30 | 31 | 36 |

Can you say that the two samples came from the same population?

(ii) The following data represent the monthly sales (in rupees) of a certain store in a leap year. Examine if there is any seasonality in the sales. 6100, 5600, 6350, 6050, 6,200, 6,300, 6,250, 5,800, 6,000, 6,150 and 6,150. (8)

15. (a) (i) State the mathematical model used in analysis of variance in a two way classification. Explain the hypothesis to be used. Discuss the advantages of this method over one way classification if any. (8)

(ii) A trucking company wishes to test the average life of each of the four brands of tyres. The company uses all brands on randomly selected trucks. The recores showing the lines (thousands of miles) of tyres are as given in the adjoining table.

| Brand I | Brand II | Brand III | Brand IV |
|---|---|---|---|
| 20 | 19 | 21 | 15 |
| 23 | 15 | 19 | 17 |
| 18 | 17 | 20 | 16 |
| 17 | 20 | 17 | 18 |
| | 16 | 16 | |

Test the hypothesis that the average life for each brand of tyres is the same. Assume $\alpha = 0.01$ $(F_{3.14}(0.01) = 5.56)$. (8)

Or

(b) (i) Explain the basic principles of experimentation. Explain how far these principles are met with in the Latin square design. (8)

(ii) A company wants to purchase cars for its own use. He has to select the make of the car out of the four makes. A, B, C and D available in the market. For this he tries five cars of each make by assigning the cars to four drivers to run on four different routes. For this, he chooses a Latin square design. The efficiency of cars is measured hi terms of time in hours. The layout and time consumed is given below.

| Routers | Drivers | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| 1 | 18(C) | 12(D) | 16(A) | 20(B) |
| 2 | 26(D) | 34(A) | 25(B) | 31(C) |
| 3 | 15(B) | 22(C) | 10(D) | 28(A) |
| 4 | 30(A) | 20(B) | 15(C) | 9(D) |

Analyse the experimental data and draw conclusions $(F_{.05}(3, 5) = 5.41)$ (8)