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## Question Paper Code : 21774

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fourth Semester

Computer Science and Engineering

MA 2262/MA 44/MA 1252/080250008/10177 PQ 401 – PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulations 2008/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Let X be the random variable which denotes the number of heads in three tosses of a fair coin. Determine the probability mass function of $X$.

2. A continuous random variable X has a probability density function given by

$$f(x)=\begin{cases}\frac{3}{4}\left(2x-x^2\right), & 0<x<2\\ 0 & \text{Otherwise}\end{cases}$$

Find $P(X>1)$.

3. Determine the value of the constant $c$ if the joint density function of two discrete random variables $X$ and $Y$ is given by $p(m,n)=cmn,\ m=1,2,3$ and $n=1,2,3$.

4. The lines of regression in a bivariate distribution are $X+9Y=7$ and $Y+4X=\frac{49}{3}$. Find the coefficient of correlation

5. Find the variance of the stationary process {X(t)}, whose auto correlation function is given by $R_{XX}(\tau)=16+\frac{9}{1+6\tau^2}$.

6. Consider a Markov chain with state {0,1,2} and transition probability matrix $P=\begin{pmatrix}0 & 1/2 & 1/2\\ 1/2 & 0 & 1/2\\ 1 & 0 & 0\end{pmatrix}$ .Draw the state transition diagram.

7. What effect does doubling $\lambda$ and $\mu$ have on $L_S$ and $W_S$ for an $(M/M/1)$: $(FIFO/\infty/\infty)$ queuing model?

8. Write the steady state probabilities for the $(M/M/R) : (GD/K/K), R \le K$ queuing model.

9. Derive the Pollaczek-Khintchine formula for the average number in the system when the service time is constant with mean $1/\mu$.

10. Distinguish between open and closed queuing network.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Ten percent of the tools produced in a certain manufacturing company turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly 2 will be defective by using

(1) binomial distribution

(2) The Poisson approximation to the binomial distribution. (8)

(ii) The number of typing mistakes that a typist makes on a given page has a Poisson distribution with a mean of 3 mistakes. What is the probability that she makes

(1) Exactly 7 mistakes

(2) Fewer than 4 mistakes

(3) No mistakes on a given page. (8)

Or

(b) (i) The lifetime $X$ of particular brand of batteries is exponentially distributed with a mean of 4 weeks. Determine

(1) The mean and variance of $X$.

(2) What is the probability that the battery life exceeds 2 weeks?

(3) Given that the battery has lasted 6 weeks, what is the probability that it will last at least another 5 weeks? (8)

(ii) Find the moment generating function of $N(\mu, \sigma^2)$ normal random variable and hence determine the mean and variance. (8)

12. (a) (i) The probability density function of $X$ and $Y$ is given by

$$f(x,y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), 0 < x < 1, 0 < y < 2$$

(1) Compute the marginal density function of $X$ and $Y$.

(2) Find $E(X)$ and $E(Y)$

(3) Find $P\left(X < \frac{1}{2}, Y > \frac{1}{2}\right)$ (10)

(ii) If $X$ and $Y$ each follow an exponential distribution with parameter 1 and are independent, find the probability density function of $U = X - Y$. (6)

Or

(b) (i) If $X, Y$ and $Z$ are uncorrelated random variables with zero means and standard deviation 5,12 and 9 respectively and if $U = X + Y$ and $V = Y + Z$, find the correlation coefficient between $U$ and $V$. (10)

(ii) If $X_1, X_2 \ldots, X_n$ are Poisson variates with parameter $\lambda = 2$,, use CLT to estimate $P(120 < S_n < 160)$ where $S_n = X_1 + X_2 + \ldots + X_n$ and $n = 75$. (6)

13. (a) (i) Show that the random process $X(t) = A\cos(wt + \theta)$ is a Wide Sense Stationary process if $A$ and $w$ are constants and $\theta$ is a uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) Let $\{X_n\}$ be a Markov chain with state space $\{0,1,2\}$ with initial probability vector $p^{(0)} = (0.7, 0.2, 0.1)$ and the one step transition probability matrix $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$

Compute $P(X_2 = 3)$ and $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$. (8)

Or

(b) (i) A random process $X(t)$ is defined by $X(t) = A\cos t + B\sin t, -\infty < t < \infty$ where $A$ and $B$ are independent random variables each of which has a value -2 with probability 1/3 and a value 1 with probability 2/3, show that $X(t)$ is a wide sense stationary process. (8)

(ii) If $\{N_1(t)\}$ and $\{N_2(t)\}$ are two independent Poisson process with parameter $\lambda_1$ and $\lambda_2$ respectively, show that $P(N_1(t) = k / N_1(t) + N_2(t) = n) = \binom{n}{k} p^k q^{n-k}$, where $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ and $q = \frac{\lambda_2}{\lambda_1 + \lambda_2}$. (8)

14. (a) (i) A car park has a capacity for 5 cars. The arrival of cars is Poisson at a mean rate of 10 per hour. The length of time each car spends in the car park has negative exponential distribution with mean of 2 minutes. How many cars are in the car park on an average and what is the probability of the newly arriving customer finding the car park full and leaving to park his car elsewhere. (8)

(ii) In a production shop of a company, the breakdown of the machines is found to be Poisson with the average rate of 3 machines per hour. Breakdown time at one machine costs Rs. 40 per hour to the company. There are two choices before the company for hiring the repairman. One of the repairman is slow but cheap, the other fast but expensive. The slow repairman demands Rs. 20 per hour and will repair the broken down machines exponentially at the rate of 4 per hour. The fast repairman demands Rs. 30 per hour and will repair the machines exponentially at an average rate of 6 per hour. Which repairman should the company hire? (8)

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(b) (i) If $X, Y$ and $Z$ are uncorrelated random variables with zero means and standard deviation 5,12 and 9 respectively and if $U = X + Y$ and $V = Y + Z$, find the correlation coefficient between $U$ and $V$. (10)

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Compute $P(X_2 = 3)$ and $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$. (8)

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Or

(b) (i) A bank has two tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles deposits only. It has been found that the service time distribution for both deposits and withdrawals are exponential with mean service time of 3 min per customer. Depositors are found to arrive in a Poisson fashion throughout the day with mean arrival rate of 16 per hour. Withdrawals also arrive in a Poisson fashion with mean arrival rate of 14 per hour. What would be the effect on the average waiting time in the queue for the customers if each teller could handle both withdrawals and deposits. What would be the effect, if this could only be accomplished by increasing the service time to 3.5 min? (8)

(ii) Consider a bank with two tellers. An average of 80 customers per hour arrive at the bank and wait in the single line for an idle teller. The average time it takes to serve a customer is 1.2 minutes. Assume that interarrival times and service times are exponential. Determine

(1) The expected number of customers present in the bank

(2) The expected length of time a customer spends in the bank

(3) The fraction of time that a particular teller is idle (8)

15. (a) Derive the Pollaczek-Khintchine formula for the average number in the system in a $M/G/1$ queueing model. (16)

Or

(b) Consider a Jackson network with parameter values shown below:

| Facility $j$ | $s_j$ | $\mu_j$ | $a_j$ | $i=1$ | $i=2$ | $i=3$ |
|---|---|---|---|---|---|---|
| $j=1$ | 1 | 10 | 1 | 0 | 0.1 | 0.4 |
| $j=2$ | 2 | 10 | 4 | 0.6 | 0 | 0.4 |
| $j=3$ | 1 | 10 | 3 | 0.3 | 0.3 | 0 |

(i) Find the steady state distribution of the number of customers at facility 1, facility 2 and facility 3.

(ii) Find the expected total number of customers in the system.

(iii) Find the expected total waiting time for a customer. (16)

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