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B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Third Semester

Civil Engineering

MA 2211/MA 31/MA 1201 A/CK 201/080100008/080210001/10177 MA 301 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS / MATHEMATICS – III

(Common to All Branches)

(Regulation 2008/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the sufficient conditions for a function to be expanded as a Fourier series.

2. Expand $f(x)=1$ in $0<x<\pi$ as a series of sines.

3. State Fourier integral theorem.

4. Find the Fourier sine transform of $f(x)=e^{-x/2}$.

5. Form the PDE by eliminating the arbitrary constants 'a', 'b' from the relation $4\left(1+a^2\right)z=(x+ay+b)^2$.

6. Solve $\left(D^3-4D^2D'+4DD'^2\right)z=0$.

7. State the assumptions in deriving the one dimensional wave equation $y_{tt}=\alpha^2 y_{xx}$.

8. Write the possible solutions of the Laplace equation $u_{xx}+u_{yy}=0$.

9. Find $Z\left[\frac{1}{n+1}\right]$.

10. State the convolution theorem on Z-transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$ and hence deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots \infty = \frac{\pi^4}{90}$. (8)

(ii) The following table gives the variations of a periodic current over a period.

t secs :	0	T/6	T/3	T/2	2T/3	5T/6	T
A amps :	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

By harmonic analysis, show that there is a direct current part of 0.75 amps in the variable current. Also obtain the amplitude of the first harmonic. (8)

Or

(b) (i) Find the half range sine series for $f(x) = \sin ax$ in $(0, l)$. (8)

(ii) Find the complex form of the Fourier series of $e^{-ax}, -l < x < l$. Deduce that when α is constant other than an integer

$$\cos \alpha x = \sin \alpha l \sum_{n=-\infty}^{\infty} \frac{\alpha l}{\alpha^2 l^2 - n^2 \pi^2} (-1)^n e^{in\pi x/l}.$$ (8)

12. (a) (i) Show that the Fourier transform of $f(x) = \begin{cases} a - |x|, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ is $\sqrt{\frac{2}{\pi}}\left(\frac{1 - \cos as}{s^2}\right)$. Hence deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$. (8)

(ii) Solve for $f(x)$, the integral equation

$$\int_0^\infty f(x) \sin sx \, dx = \begin{cases} 1, & 0 \le s < 1 \\ 2, & 1 \le s < 2 \\ 0, & s \ge 2 \end{cases}$$ (8)

Or

(b) (i) Find the Fourier transform of $e^{-|x|}$ and hence deduce that $\int_0^\infty \frac{\cos xt}{1+t^2} dt = \frac{\pi}{2} e^{-|x|}$. (8)

(ii) Prove that $F_C[x f(x)] = \frac{d}{ds}[F_S\{f(x)\}]$ and $F_S[x f(x)] = -\frac{d}{ds}[F_C\{f(x)\}]$ (8)

13. (a) (i) Form the PDE by eliminating the arbitrary functions f_1, f_2 from the relation $z = x f_1(x+t) + f_2(x+t)$. (8)

(ii) Solve $\left(\frac{p}{2}+x\right)^2 + \left(\frac{q}{2}+y\right)^2 = 1$. (8)

Or

(b) (i) Solve $x^2 p + y^2 q = z(x+y)$. (8)

(ii) Solve $(r + s - 6t) = y \cos x$. (8)

14. (a) An uniform elastic string of length 60 cms is subjected to a constant tension of 2 Kg. If the ends fixed and the initial displacement $y(x,0) = 60x - x^2$, $0 < x < 60$, while the initial velocity is zero, find the displacement function $y(x,t)$. (16)

Or

(b) Solve the problem of heat conduction in a rod given that the temperature function $u(x,t)$ is subject to the condition, $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le l$, $t > 0$

(i) u is finite as $t \to \infty$

(ii) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = l$, $t > 0$

(iii) $u = lx - x^2$ for $t = 0$, $0 \le x \le l$. (16)

15. (a) (i) Find $Z(r^n \sin n\theta)$, $Z^{-1}\left[\frac{z}{z^2+4z+3}\right]$. (4+4)

(ii) Find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ using convolution theorem. (8)

Or

(b) (i) Using complex residue theorem evaluate $Z^{-1}\left[\frac{9z^3}{(3z-1)^2(z-2)}\right]$. (8)

(ii) Solve using Z-transforms technique the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0$, $y_1 = 1$. (8)
