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Reg. No. :

Question Paper Code : 71771

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 — MATHEMATICS — II

(Common to All Branches)

(Regulation 2008)

Time : 3 hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Solve : $(D^2 - 2D + 5)y = 0$.

2. Find the particular integral of $(D^3 + 4D)\,y = \sin 2x$.

3. Evaluate : $\nabla(r^n)$ where $r^2 = x^2 + y^2 + z^2$.

4. Find a such that $\vec{F} = (3x - 2y + z)\,\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.

5. Show that the function $v(x, y) = -\sin x \sinh y$ is a harmonic function.

6. Find the invariant points of the transformation $w = \dfrac{1 + iz}{1 - iz}$.

7. State Cauchy's integral theorem.

8. Identify the type of the singularity of the function $\dfrac{1}{\cos z - \sin z}$.

9. Find the Laplace transform of unit step function.

10. Evaluate $\int_0^\infty e^{-t} \cos t \; dt$ using Laplace transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve : $(D^2 - 4D + 3)\,y = e^x \cos 2x$. (8)

(ii) Solve : $(x^3D^3 + 3x^2D^2 + xD + 1)\,y = x + \log x$. (8)

Or

(b) (i) Solve the simultaneous differential equations :

$\frac{dx}{dt} + 5x - 2y = t$ and $\frac{dy}{dt} + 2x + y = 0$. (8)

(ii) Solve the differential equation $(D^2 + 1)y = \operatorname{cosec} x \cot x$ using the method of variation of parameters. (8)

12. (a) (i) Prove that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational. Hence find its scalar potential φ. (8)

(ii) Using Green's theorem, evaluate $\int_C (x^2 - 2xy)\,dx + (x^2y + 3)\,dy$, where C is the region bounded by the curves $y^2 = 8x$ and $x = 2$. (8)

Or

(b) Verify Gauss divergence theorem for $\vec{F} = (4xz)\vec{i} - (y^2)\vec{j} + (yz)\vec{k}$ taken over the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (16)

13. (a) (i) Determine the analytic function $w = u + iv$ if $u = \frac{\sin 2x}{(\cosh 2y - \cos 2x)}$. (8)

(ii) If $f(z)$ is a regular function, show that $\left[\frac{\partial |f|}{\partial x}\right]^2 + \left[\frac{\partial |f|}{\partial y}\right]^2 = |f'|^2$. (8)

Or

(b) (i) Find the image of the infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. (8)

(ii) Find the bilinear transformation that maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$ respectively. (8)

14. (a) (i) Evaluate : $\int_C \frac{\sin^2 z}{\left(z-\frac{\pi}{6}\right)^3} dz$, where C is $|z|=1$. (6)

(ii) Find the Laurent's series expansion of $f(z)=\frac{7z-2}{(z+1)z(z-2)}$ valid in the region $1<|z+1|<3$. (10)

Or

(b) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$, using contour integration. (16)

15. (a) (i) Find : $L\{te^{-t}\sin 3t\}$. (4)

(ii) Find : $L\left\{\frac{\cos at-\cos bt}{t}\right\}$. (4)

(iii) Find : $L^{-1}\left\{\frac{s^2}{(s^2+a^2)\,(s^2+b^2)}\right\}$. (8)

Or

(b) (i) Find the Laplace transform of the square wave function $f(t)$ defined by $f(t)=\begin{cases} k & \text{in } 0\le t\le a \\ -k & \text{in } a\le t\le 2a \end{cases}$ and $f(t+2a)=f(t)$ for all t.

(8)

(ii) Solve the differential equation $y''+9y=\cos 2t$, where $y(0)=1$ and $y\left(\frac{\pi}{2}\right)=-1$ using Laplace transforms. (8)