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## Question Paper Code : 91021

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Second Semester

Civil Engineering

MA 205 — MATHEMATICS — II

(Common to All Branches)

(Regulation 2007)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Change the order of integration $\int_0^1 \int_0^x f(x, y) dy\, dx$.

2. Find the area of the circle $x^2 + y^2 = a^2$, by using double integration.

3. Find the value of $a$, $b$, $c$ so that the vector $\vec{F} = (x + y + az)\vec{i} + (bx + 2y - z)\vec{j} + (-x + cy + 2z)\vec{k}$ may be irrotational.

4. Show that force $\vec{F} = (2x + yz)\vec{i} + (xz - 3)\vec{j} + xy\vec{k}$ is conservative.

5. State any two properties of an analytic function.

6. Define critical point of a transformation.

7. Evaluate $\oint \frac{z}{(z-2)^3} dz$ around the closed curve $|z| = 1$.

8. Classify the singularity of $e^{1/z}$.

9. Find $L^{-1}\left[\frac{s}{(s+2)^3}\right]$.

10. Show that $L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$ where $F(s) = L\{f(t)\}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Evaluate $\int_{0}^{4}\int_{y^2/4}^{y} \frac{y}{x^2 + y^2} dx$. (8)

(ii) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration. (8)

Or

(b) (i) Find the area of the cardioid $r = a(1 + \cos\theta)$. (8)

(ii) Evaluate $\int_{0}^{\infty}\int_{0}^{x} xe^{x^2/y}\, dy\, dx$ by using the change of order of integration. (8)

12. (a) (i) Find the directional derivative of $\phi = 2xy + z^2$ at $(1, -1, 2)$ in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$. (6)

(ii) Use divergence theorem to evaluate $\iint \left(4x\,\vec{i} - 2y^2\,\vec{j} + z^2\,\vec{k}\right)$ over the closed surface bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 3$. (10)

Or

(b) (i) Use Green's theorem in a plane to evaluate $\int \left[x^2(1 + y)dx + \left(x^3 + y^3\right)dy\right]$ around the square bounded the lines $x = \pm 1$ and $y = \pm 1$. (8)

(ii) Use Stoke's theorem to find the value of $\oint_C \vec{F} \cdot \vec{dr}$, when $\vec{F} = \left(xy - x^2\right)\vec{i} + x^2 y\,\vec{j}$ and $C$ is the closed curve boundary of the triangle in the $xoy$ plane formed by $x = 1$, $y = 0$ and $y = x$. (8)

13. (a) (i) Find the analytic function $w = u + iv$ if $v = e^{-2y}\left(y \cos 2x + x \sin 2x\right)$. Hence find $u$. (8)

(ii) Show that the transformation $w = \frac{1}{z}$ maps the circle $|z - 3| = 5$ onto the circle $\left|w + \frac{3}{16}\right| = \frac{5}{16}$. What is the image of the interior of the given circle in the $z$ plane? (8)

Or

(b) (i) Find the bilinear transformation which maps the points $i, -1, 1$ of the $z$ plane into the points $0, 1, \infty$ of the $w$ plane respectively. (8)

(ii) Verify that the family of curves $u = c_1$ and $v = c_2$ cut orthogonally when $w = u + iv = z^3$. (8)

14. (a) (i) Evaluate $\int_0^{2\pi} \frac{\sin^2\theta}{5 - 3\cos\theta}$ using contour integration. (8)

(ii) Find the Laurent's series of $f(z) = \frac{z}{(z-1)(z-2)}$ valid in the region $3 < |z+2| < 4$. (8)

Or

(b) (i) Use Cauchy's integral formula to evaluate $\int \frac{\sin \pi z + \cos \pi z}{(z-2)(z-3)} dz$ over the circle $|z| = 4$. (8)

(ii) Evaluate $\int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ by contour integration technique. (8)

15. (a) (i) Find $L\left[\int_0^1 \frac{1 - 2\cos t}{t} dt\right]$. (8)

(ii) Use Laplace transform to :
$y'' + y' - 2 = 3\cos 3t - 11\sin 3t, y(0) = 0, y'(0) = 6$. (8)

Or

(b) (i) Use convolution theorem to find the inverse Laplace transform of $\frac{4}{(s^2+2s+5)^2}$. (8)

(ii) Find the Laplace transform of $f(t) = |\sin wt|, t \geq 0$. (8)

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