Reg. No. :

Question Paper Code : 73529

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Third Semester

Civil Engineering

MA 1201/070030007/070030005/070030004 — MATHEMATICS — III

(Common to all Branches)

(Regulation 2004/2007)

(Common to B.E. (Part-Time) Second Semester, Civil Engineering, Computer Science and Engineering, Electrical and Electronics Engineering, Electronics and Communication Engineering and Mechanical Engineering, Regulation 2005)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating the function ϕ from $z = \phi(xy)$.

2. Find the general solution of the Lagrange linear equation given by $pyz + qzx = xy$.

3. Find a_n in the Fourier series expansion of $f(x) = x + x^3$; $(-l, l)$.

4. Find the root mean square value of the function $f(x) = x$ in $(0, l)$.

5. Classify the partial differential equation $u_{xx} + xu_{yy} = 0$.

6. Give the three possible solutions of the one dimensional heat flow equation.

7. Prove that the FCT of $\{f(x)\cos ax\}$ is given by $\frac{1}{2}[F_c(s+a) + F_c(s-a)]$ where $F_c(s)$ is the Fourier cosine transform of $f(x)$.

8. Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 0, & x < a \\ 1, & a < x < b \\ 0, & x > b. \end{cases}$

9. Find $z\left[\frac{1}{n+1}\right]$.

10. If $z\{f(n)\} = F(z)$, then show that $z[f(n)] = -z\frac{d}{dz}[F(z)]$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Form the PDE from $f(x^2 + y^2 + z^2, z^2 - xy) = 0$. (8)

(ii) Find the singular integral of $z = px + qy + p^2 - q^2$. (8)

Or

(b) (i) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. (8)

(ii) Solve $(D^3 - 3D^2D' - 4DD'^2 + 12D'^3)z = \sin(x + 2y) + e^{x+y}$. (8)

12. (a) Find the Fourier series expansion of the periodic function $f(x)$ of the period 2 defined by

$$f(x) = \begin{cases} 1+x, & -1 \le x \le 0 \\ 1-x, & 0 \le x \le 1. \end{cases}$$

Deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. (16)

Or

(b) Find the first three harmonic of Fourier series of $y = f(x)$ from the following data : (16)

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
y	298	356	373	337	254	155	80	51	60	93	147	221

13. (a) A string is stretched and fastened to two points $x = 0$ and $x = l$ apart. The $\frac{l}{3}$ point of the string is displaced to a distance 'h' and is released from rest. Find the displacement at any point of the string at any time t. (16)

Or

(b) The ends A and B of a rod of 30 cms length have their temperature kept at 20°C and the other at 80°C, until steady conditions prevail. The temperature of the end B is suddenly reduced to 60°C and kept so while the end A is raised to 40°C. Find the temperature distribution in the rod after time t. (16)

14. (a) Find the Fourier transform of

$$f(x) = \begin{cases} a^2 - x^2; & |x| < a \\ 0; & |x| > a,\ a > 0 \end{cases}$$

and hence deduce

(i) $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3} \right) dt$

(ii) $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3} \right)^2 dt$. (16)

Or

(b) (i) Find the Fourier sine and cosine transforms of xe^{-ax}, $a > 0$. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ using Fourier sine or cosine transforms. (8)

15. (a) (i) Find the response of the system $y_{n+2} - 5y_{n+1} + 6y_n = u_n$ with $y_0 = 0$, $y_1 = 1$ and $u_n = 1$ for $n = 0, 1, 2, 3, \ldots$ by z-transform method. (8)

(ii) Find the inverse z-transform of $\frac{z^3 - 20z}{(z-2)^3(z-4)}$. (8)

Or

(b) (i) Use convolution theorem to evaluate $z^{-1}\left\{ \frac{z^2}{(z-a)(z-b)} \right\}$. (8)

(ii) If $z[n^2] = \frac{z^2 + z}{(z-1)^3}$ then find $z[(n+1)^2]$. (8)

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