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Question Paper Code : 21771

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 – MATHEMATICS – II

(Common to all Branches)

(Regulations 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Transform $(x^2D^2 - 3xD + 2)y = \sin(2\log x)$ into an ordinary differential equation with constant coefficient.

2. Find a differential equation of $x(t)$ given $\frac{dy}{dt} + x = \cos t; \frac{dx}{dt} + y = e^{-t}$.

3. Prove that $\nabla r^n = nr^{n-2}\vec{r}$

4. Using Green's theorem in a plane show that the area enclosed by a simple closed curve C is $\frac{1}{2}\int_C xdy - ydx$.

5. Prove that a real part of an analytic function is a harmonic function.

6. Find the invariant points of $w = \frac{z}{z^2 - 2}$.

7. Evaluate $\int_C \frac{e^{3z}}{z(z-1)} dz$ where C is the circle $|z - 3| = 1$.

8. Define essential singularity and give example.

9. What is meant by exponentially ordered function?

10. Evaluate $\int_0^\infty \frac{1 - e^{-t}}{t} dt$ by using Laplace Transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve: $\frac{d^2y}{dx^2} - 11\frac{dy}{dx} + 18y = e^{+2x} + e^{-x}\cos x + x$. (8)

(ii) Solve: $\frac{dx}{dt} + 2x - 3y = t; \frac{dy}{dt} - 3x + 2y = e^{2t}$. (8)

Or

(b) (i) Solve $\frac{d^2y}{dx^2} + 4y = \cot 2x$ by using variation of parameters. (8)

(ii) Solve $(7+2x)^2\frac{d^2y}{dx^2} - 6(7+2x)\frac{dy}{dx} + 8y = 6x$. (8)

12. (a) (i) Find the angle between the surfaces $x^2 - y^2 - z^2 = 11$ and $xy + yz + zx - 18 = 0$ at the point (6,4,3). (6)

(ii) Verify Gauss Divergence Theorem for $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2, z = 0$ and $z = h$. (10)

Or

(b) (i) Prove that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is a conservative field hence find the scalar potential of $\vec{F}$. (6)

(ii) Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ over the box bounded by the planes $x = 0, x = a, y = 0, y = b, z = 0, z = c$ if the face $z = 0$ is cut. (10)

13. (a) (i) If $u(x,y)$ and $v(x,y)$ are harmonic functions in a region R, prove that $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is analytic. (8)

(ii) Find the bilinear transformation which transform the points $z = 0, 1, \infty$ into $w = i, -1, -i$ respectively. (8)

Or

(b) (i) If $f(z) = u + iv$ is analytic, find $f(z)$ given that $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$. (8)

(ii) Under the transformation $w = \frac{1}{z}$ determine the region in w – plane of the infinite strip bounded by $\frac{1}{4} \le y \le \frac{1}{2}$. (8)

14. (a) (i) Evaluate $\int_C \frac{e^z dz}{z(1-z)^3}$ if C is $|z| = 2$, by using Cauchy's integral formula. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{x^4 + a^4}$. (8)

Or

(b) (i) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series valid in $1 < |z| < 3$ and also $0 < |z+1| < 2$. (8)

(ii) By using Cauchy's residue theorem evaluate $\int_C \frac{\sin \pi z + \cos \pi z}{(z+2)(z+1)^2} dz$ where C is $|z| = 3$. (8)

15. (a) (i) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ and $f(t + 2a) = f(t)$. (5)

(ii) Find the inverse Laplace transform of $\frac{se^{-s}}{s^2 + 4}$. (4)

(iii) Solve $y'' + 2y' + y = te^{-t}, y(0) = 1, y'(0) = -2$. (7)

Or

(b) (i) Define unit impulse function and also find its Laplace transform. (6)

(ii) Using convolution theorem find inverse Laplace transform of $\frac{s^2 + s}{(s^2 + 4)(s^2 + 2s + 10)}$. (10)
