Reg. No. :

Question Paper Code : 13021

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Second Semester

Civil Engineering

MA 205 — MATHEMATICS — II

(Common to All branches)

(Regulation 2007)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Draw the region of integration $\int_0^1 \int_0^{1-x} f(x, y)\, dx\, dy$.

2. Find the value of the Jacobian given that $x = r \sin\theta \cos\varphi$; $y = r \sin\theta \sin\varphi$ and $z = r \sin\theta$.

3. Find $div\left(\frac{\bar{r}}{r}\right)$.

4. Evaluate $\int_C x\,dy - y\,dx$ where C is $x^2 + y^2 = 4$.

5. Verify that $e^y \cosh x$ is a harmonic function.

6. Find the invariant points of $\omega = \frac{2z+6}{z+7}$.

7. Evaluate $\oint_C [\text{Re}(z) + z]\, dz$ where $C : |z| = 2$.

8. Classify the singular point of $\frac{e^z}{(z - \sin z)}$.

9. If $L\{f(t)\} = \frac{1}{s(s+2)}$, find $\lim_{t \to \infty} f(t)$.

10. Find the inverse Laplace transform of $\frac{1}{(s+2)^3}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 5$. (8)

(ii) Transform the double integral $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dxdy}{\sqrt{a^2 - x^2 - y^2}}$ into polar co-ordinates and hence evaluate it. (8)

Or

(b) (i) Change the order of integration and then evaluate $\int_0^{2a} \int_{\frac{x^2}{4a}}^{a} (x+y)\, dy\, dx$. (8)

(ii) Find the area that lies outside the circle $r = 3a\cos\theta$ and outside the cardioid $r = a(1+\cos\theta)$ using double integration. (8)

12. (a) (i) Find the directional derivative of $\phi = 4xz^2 + x^2yz$ at $(1, -2, 1)$ in the direction of $2\vec{i} + 3\vec{j} + 4\vec{k}$. (4)

(ii) Verify Stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the circular boundary on the plane $z = 0$. (12)

Or

(b) (i) Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is irrotational. Find its scalar potential ϕ such that $\vec{F} = \nabla\phi$. (8)

(ii) Verify Green's theorem in the plane for $\int_C (3x^2 - 8y^2)\, dx + (4y - 6xy) dy$ where C is the boundary of the triangle formed by the lines $x = 0,\ y = 0,\ x + y = 1$. (8)

13. (a) (i) If ϕ and ψ are the function of x and y satisfying Laplace equation show that $\left(\frac{\partial\varphi}{\partial y}-\frac{\partial\psi}{\partial x}\right)+i\left(\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial y}\right)$ is analytic. (8)

(ii) Find the bilinear transformation which maps $z = 1, i = -1$ into the planes $w = i, 0, -i$. Find the image of $|z| < 1$. (8)

Or

(b) (i) Determine the analytic function $f(z) = u + iv$ given $v = \log(x^2 + y^2) + x - 2y$. (8)

(ii) Consider the map $w = 1/z$ and determine the region R' in ω - plane of the infinite strip R bounded by $0.25 < y < 0.5$. (8)

14. (a) (i) Evaluate $\int_C \frac{e^z\,dz}{(z+2)(z+1)^2}$ where C is the circle $|z| = 2$, using Cauchy's integral formula. (8)

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+a^2)(x^2+b^2)}$ by using contour integration where $a > b > 0$. (8)

Or

(b) (i) Expand $\frac{1}{z^2-3z+2}$ in Laurent's series valid in the regions $1 < |z| < 2$ and $2 < |z+1| < 3$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta\, d\theta}{5+4\cos\theta}$ using contour integration. (8)

15. (a) (i) Find the Laplace transform of $\frac{e^{at}-\cos bt}{t}$. (4)

(ii) Find the inverse Laplace transform of $\tan^{-1}(2/s^2)$. (4)

(iii) Solve $(D^3 + D)x = 2$ given $x(0) = 3;\; x'(0) = 1;\; x''(0) = -2$. (8)

Or

(b) (i) Find the Laplace transform of $f(t) = \frac{t}{p}$, $0 < t < p$ and $f(t+p) = f(t)$. (6)

(ii) Using convolution theorem find $L^{-1}\left(\frac{s}{(s^2+1)(s^2+9)}\right)$. (6)

(iii) Solve $y(t) = a\sin t - 2\int_0^t y(u)\cos(t-u)\,du$. (4)
