Reg. No. :

Question Paper Code : 13020

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

First Semester

Civil Engineering

MA 105 — MATHEMATICS – I

(Common to all Branches)

(Regulation 2007)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If two of the eigen values of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ are 2 and 8, what is the third eigen value?

2. Discuss the nature of the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4zx - 8yz$.

3. Find the centre and raditis of the sphere $x^2 + y^2 + z^2 + 2y - 4z - 4 = 0$.

4. If $\frac{x}{1} = \frac{y}{2} = \frac{z}{k}$ a generator of the cone $x^2 + y^2 - z^2 = 0$, find the value of k.

5. Find the curvature of the circle $x^2 + y^2 = 25$ at $(3,4)$.

6. Find the equation of the envelope of the family of curves $x^2 + y^2 - 2px + p^2/2 = 0$.

7. If $f(x,y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{(\log x - \log y)}{x^2 + y^2}$, prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 2f = 0$.

8. Let $f(x,y) = x \sin y + e^x \cos y$, $x = t^2 + 1$, $y = t^2$ then find the value of $\left(\frac{\partial f}{\partial t}\right)_{t=0}$.

9. Solve $(D^2+1)^2 . y=0$, where $D=\frac{d}{dx}$.

10. Find the particular integral of $\frac{d^2y}{dx^2}-4y=\cosh(2x-1)$.

PART B — (5 × 16 = 80 marks)

11. (a) Verify Cayley—Hamilton theorem for the matrix $A=\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

Hence compute A^{-1}.

Or

(b) Reduce $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_3x_1$ into canonical form by an orthogonal transformation.

12. (a) Find the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Find also the equations and the points in which the S.D meets the given lines.

Or

(b) Find the equation of the right circular cone generated by the straight lines drawn from the origin to cut the circle through the three points $(1,2,2)$, $(2,1,-2)$ and $(2,-2,1)$.

13. (a) (i) Find the radius of curvature for $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $(a/4, a/4)$. (8)

(ii) Find the evolute of the parabola $x^2=4y$. (8)

Or

(b) (i) Obtain the envelope of $y=mx-2am-am^3$ where m is the parameter. (8)

(ii) Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by considering it as the envelope of normals. (8)

14. (a) (i) Discuss the maxima and minima of $x^3 y^2 (1-x-y)$. (10)

(ii) Expand $e^x \sin y$ in powers of x and y as far as terms of second degree. (6)

Or

(b) (i) Find the minimum value of $x^2 + y^2 + z^2$, given that $ax + by + cz = p$. (10)

(ii) If $u = xyz$, $v = x^2 + y^2 + z^2$, $w = x + y + z$, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. (6)

15. (a) (i) Solve $(x^2 D^2 - 3xD + 4)y = x^2 + \cos(\log x)$. (8)

(ii) Solve $\frac{dx}{dt} + 2y = \sin 2t$, $\frac{dy}{dt} - 2x = \cos 2t$. (8)

Or

(b) (i) Solve by the method of reduction of order $(x+1)\frac{d^2 y}{dx^2} - 2(x+3)\frac{dy}{dx} + (x+5)y = e^x$. (8)

(ii) Solve $(D^2 + 2D + 1)y = x^3 + \sin 2x \cos x$. (8)

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