

9. Express $\int_0^a \int_0^a \frac{x^2}{\sqrt{x^2+y^2}} dx dy$ into polar coordinates.

10. Evaluate : $\int_0^2 \int_0^y \int_0^x dx dy dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Verify the Cayley-Hamilton theorem for a matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and hence find A^{-1} . (8)
- (ii) Find the eigenvalues and eigenvectors of a matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (8)

Or

- (b) Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$ to the canonical form through an orthogonal transformation. Hence find the following :
- (i) Nature of the quadratic form
- (ii) Rank, index and signature of the quadratic form, and
- (iii) A set of non-zero values of x, y, z which will make the quadratic form zero. (16)

12. (a) (i) Find the equation of the smallest sphere which contains the circle given by the equations $x^2 + y^2 + z^2 + 2x + 4y + 6z - 11 = 0$ and $2x + y + 2z + 1 = 0$. (8)
- (ii) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B and C . Find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC . (8)

Or

- (b) (i) Find the centre and radius of the circle given by $x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0$ and $x + 2y + 2z - 20 = 0$. (8)
- (ii) Find the equation of the right circular cylinder of radius 3 and whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. (8)

13. (a) (i) Find the radius of curvature of the curve $y^2 = \frac{(a^3 - x^3)}{x}$ at the point $(a, 0)$. (8)

(ii) Find the envelope of the straight line $y \cos \theta - x \sin \theta = a \cos 2\theta$, θ being the parameter. (8)

Or

(b) (i) Find the equation of the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = 2$ at the point $(1, 1)$. (8)

(ii) Find the equation of the evolute of the curve $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$. (8)

14. (a) (i) If $u = \tan^{-1} \left[\frac{x^3 + y^3}{x - y} \right]$, using Euler's theorem on homogeneous functions, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$. (8)

(ii) Find the maximum and minimum values of

$$f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x. \quad (8)$$

Or

(b) (i) Obtain the Taylor's series expansion of $x^3 + 4x^2y - 2xy^2 + y^3$ near the point $(-1, 1)$ upto the third degree terms. (8)

(ii) A rectangular box, open at the top, is to have a volume of 108 c.c. Find the dimensions of the box that requires the least material for its construction. (8)

15. (a) (i) Change the order of integration $\int_0^{1-x} \int_{x^2}^{2-x} xy \, dx \, dy$ and hence evaluate. (8)

(ii) Find the area that lies outside the circle $r = 2 \cos \theta$ and inside the circle $r = 6 \cos \theta$, using double integration. (8)

Or

(b) (i) Find the volume of the cylinder $x^2 + y^2 = 25$ bounded by the planes $z = 1$ and $x + z = 10$. (8)

(ii) Evaluate $\iint_R \frac{xy \, dx \, dy}{\sqrt{x^2 + y^2}}$ where R is the region in the first quadrant enclosed by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 16$. (8)