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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fifth/Third Semester

Civil Engineering

MA 2211/MA 31/MA 1201 A/CK 201/080100008/080210001/10177 MA 301 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS/ MATHEMATICS – III

(Common to all branches)

(Regulations 2008/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the function $f(x) = x$ in the interval $0 < x < 2\pi$, then find the constant term of the Fourier series expansion of the function f.

2. If the Fourier series expansion of the function $f(x) = \begin{cases} -k, -\pi < x < 0; \\ k, \ 0 < x < \pi \end{cases}$ is $\sum_{n=1,3,5,\ldots}^{\infty} \frac{4k \sin nx}{n\pi}$, then find the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\infty$.

3. Find the Fourier cosine transform of $f(x) = e^{-ax}, a > 0$.

4. Prove that Fourier transform is linear.

5. Form the partial differential equation by eliminating a and b from $z = a^2x + ay^2 + b$.

6. Solve : $pq = xy$.

7. Write down the initial conditions when a taut string of length $2l$ is fastened on both ends. The midpoint of the string is taken to a height b and released from the rest in that position.

8. The ends of A and B of a rod of length 10 cm long have their temperature kept 20°C and 70°C. Find the steady state temperature distribution on the rod.

9. Form the difference equation by eliminating arbitrary constants from $u_n = a2^{n+1}$.

10. Find $Z[e^t \sin 2t]$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the Fourier series for $y = x^2$ in $-\pi < x < \pi$. Hence show that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\infty = \frac{\pi^4}{90}$. (8)

(ii) Obtain the Fourier expansion of $x \sin x$ as a cosine series in $(0, \pi)$. Hence show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\infty = \frac{\pi - 2}{4}$. (8)

Or

(b) (i) Find the complex form of the Fourier series for the function $f(x) = e^{ax}$ in $-\pi < x < \pi$, where a is a real constant. (8)

(ii) Obtain the first three coefficients in the Fourier cosine series for y, where y is given in the following table : (8)

x	0	1	2	3	4	5
y	4	8	15	7	6	2

12. (a) (i) Express the function $f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$; as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin \mu \cos \mu x}{\mu} d\mu$ and $\int_0^\infty \frac{\sin \mu}{\mu} d\mu$. (8)

(ii) Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| < a; \\ 0, & |x| > a > 0 \end{cases}$. Hence prove that $\int_0^\infty \frac{\sin x - x \cos x}{x^3} dx = \frac{\pi}{4}$. (8)

Or

(b) (i) Solve the integral equation $\int_0^\infty f(x) \sin tx \, dx = \begin{cases} 1, & 0 \leq t < 1 \\ 2, & 1 \leq t < 2. \\ 0; & t \geq 2 \end{cases}$ (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)} dx$ using Fourier transform technique. (8)

13. (a) (i) Solve : $x^2p^2 + y^2q^2 = z^2$. (8)

(ii) Solve : $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. (8)

Or

(b) (i) Find the singular solution of $z = px + qy + \sqrt{p^2 + q^2 + 1}$. (8)

(ii) Solve : $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$. (8)

14. (a) A tightly stretched string of length l has its ends fastened at $x = 0$, $x = l$. At $t = 0$, the string is in the form $f(x) = kx(1 - x)$ and then released. Find the displacement at any point on the string at a distance x from one end at any time $t > 0$. (16)

Or

(b) A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at short edge $y = 0$ is given by

$$u = \begin{cases} 20x, & 0 \le x \le 5; \\ 20(10 - x), & 5 \le x \le 10 \end{cases}$$

and all the other three edges are kept at 0°c. Find the steady state temperature at any point in the plate. (16)

15. (a) (i) Find $Z\left[\frac{2n+3}{(n+1)(n+2)}\right]$. (8)

(ii) If $Z(u_n) = U(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$, evaluate u_2 and u_3. (8)

Or

(b) (i) Find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ using convolution theorem. (8)

(ii) Using Z-transform solve $y_{n+2} + 4y_{n+1} + 3y_n = 2^n$ with $y_0 = 0$ and $y_1 = 1$. (8)
