**Reg. No. :** | | | | | | | | | | |

**Question Paper Code: 41001**

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2015.

Fourth Semester

Computer Science and Engineering

01UMA421 - APPLIED STATISTICS & QUEUEING NETWORKS

(Common to Information Technology)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

(Statistical table is permitted)

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. State the axioms of probability.
2. If a random variable '*X*' has a uniform distribution in (-3, 3), find $P(X < 2)$.
3. If two random variable *X* and *Y* have pdf $f(x, y) = ke^{-(2x+y)}$ for $x, y > 0$. Evaluate *k*
4. State Liapounoff's form of Central limit theorem.
5. Compare RBD and LSD.
6. Write down the basic principles of experimental design.
7. Write down the Little's formula for $(M/M/1) : (\infty / FIFO)$ model.
8. If $\lambda = 4$ per hour and $\mu = 12$ per hour in an $(M / M / I) : (4 / FIFO)$ queuing system, find the probability that there is no customer in the system.
9. Define series queues. Give examples.
10. Define Open and Closed queueing networks.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Three machines $A$, $B$ and $C$ with capacities proportional to 4:2:3 are producing identical items. The percentage that the machine produce defectives are 4%, 3% and 5% respectively. At the end of a day from the total production one item is selected at random and is found defective. What is the chance that it came from machine $B$? (8)

(ii) Derive MGF, mean and variance of Geometric distribution. (8)

Or

(b) (i) The pdf of $f(x)=\begin{cases} kx, & 0 \le x \le 2 \\ 2k, & 2 \le x \le 4 \\ k(6-x), & 4 \le x \le 6 \end{cases}$

Find (1) the value of $k$

(2) the distribution function

(3) $P(X > 3)$

(4) $P(4 \le X < 5 / X > 3)$. (8)

(ii) The mileage which the car owners get with a certain kind of radial tyre is a random variable having an exponential distribution with mean 40,000 km. Find the probabilities that one of these tyres will last

(1) at least 20,000 km

(2) at most 30,000 km

(3) at least 30,000 km given that it has been used for 20,000 km. (8)

12. (a) (i) For the bivariate probability distribution of $(X,Y)$ given below, find $P(X \le 1)$, $P(Y \le 3)$, $P(X \le 1 / Y \le 3)$ and $P(X + Y \le 4)$. (8)

| X \ Y | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | $\frac{1}{32}$ | $\frac{2}{32}$ | $\frac{2}{32}$ | $\frac{3}{32}$ |
| 1 | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| 2 | $\frac{1}{32}$ | $\frac{1}{32}$ | $\frac{1}{64}$ | $\frac{1}{64}$ | 0 | $\frac{2}{64}$ |

(ii) Given the following joint density function

$$f(x,y) = \begin{cases} \frac{8}{k} xy, & 0 \le x \le y \le 2 \\ 0, & otherwise \end{cases}$$

Find (1) value of $k$, (2) marginal density functions,
(3) $P(X \le 1/Y < 3/2)$, (4) $P(X + Y \le 1)$ (8)

Or

(b) (i) Let the random variables $X$ and $Y$ have the joint probability density function

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2 + y^2) & ,0 \le x \le 1, 0 \le y \le 1 \\ 0 & ,otherwise \end{cases}$$

Compute the correlation coefficient between $X$ and $Y$. (8)

(ii) A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using central limit theorem, with what probability can we assert that the mean of the sample will not differ from $\mu = 60$ by more than 4? (8)

13. (a) A car rental agency, which uses 5 different brands of tyres in the process of deciding the brand of tyre to purchase as standard equipment for its fleet, finds that each of 5 tyres of each brand last the following number of kilometers (in thousands):

**Tyre brands**

| **A** | **B** | **C** | **D** | **E** |
|---|---|---|---|---|
| 36 | 46 | 35 | 45 | 41 |
| 37 | 39 | 42 | 36 | 39 |
| 42 | 35 | 37 | 39 | 37 |
| 38 | 37 | 43 | 35 | 35 |
| 47 | 43 | 38 | 32 | 38 |

Test the hypothesis that the five tyre brands have almost the same average life. (16)

Or

(b) Analyse the variance in the following Latin square of yields (in kgs) of paddy where *A,B,C,D* denote the different methods of cultivation

| | | | |
|---|---|---|---|
| D122 | A121 | C123 | B122 |
| B124 | C123 | A122 | D125 |
| A120 | B119 | D120 | C121 |
| C122 | D123 | B121 | A122 |

Examine whether the different methods of cultivation have given significantly different yields. (16)

14. (a) A departmental store has a single cashier. During the rush hours, customers arrive at a rate of 20 customers per hour. The cashier takes on an average 2.5 minutes per customer for processing.

(i) What is the probability that the cashier is idle and a customer shall have to wait in the queue?

(ii) What is average number of customers and average time spent by a customer in the system?

(iii) What is the average queue length and average time a customer spends in the queue?

(iv) What is the expected number of customers in a queue, if it exists?

(v) Find the average waiting time of a customer in the queue, if he has to wait. (16)

Or

(b) Honda auto service station has 5 mechanics, each of whom can service a motorbike in 2 hours on an average. The motorbikes are registered at a single counter and then sent for servicing to different mechanics. Motorbikes arrive at the service station at an average rate of 2 per hour. Determine

(i) Probability that the system shall be idle,

(ii) Probability that there shall be 3 and 8 motorbikes in the station,

(iii) Expected number of motorbikes in the service station and queue,

(iv) Average waiting time in the queue,

(v) Average time spent by a motorbike in waiting and getting serviced. (16)

15. (a) Derive Pollaczek - Khinchine formula of *M/G/1* queue. (16)

Or

(b) In a computer programs for execution arrive according to poission law with a mean of 5 per min. Assume that the system is busy. The service time is

(i) uniform between 8 and 12 seconds,

(ii) a discrete distribution with values equal to 2,7,12 seconds and corresponding probabilities 0.2, 0.5 and 0.3. Find $L_s$ , $L_q$ , $W_s$ , $W_q$. (16)

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