**Reg. No. :** | | | | | | | | | | |

**Question Paper Code: 21002**

B.E. / B.Tech. DEGREE EXAMINATION, OCTOBER 2014.

Second Semester

Civil Engineering

01UMA202 – ENGINEERING MATHEMATICS - II

(Common to all branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Solve $D^2 - 6D + 13y = 0$.

2. Transform the equation $(x^2D^2 + xD)y = x$ into a linear differential equation with constant coefficients.

3. Find a unit vector normal to the surface $x^2y + 2xz = 4$ at the point (2, -2, 3).

4. Find 'a' such that $\vec{F} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$ is solenoidal.

5. Test the analyticity of the function $f(z) = \bar{z}$.

6. Find the invariant points of the transformation $w = \dfrac{2z+6}{z+7}$.

7. State Cauchy's integral formula for first derivative of an analytic function.

8. Find the Taylor's series for $f(z) = \sin z$ about $z = \dfrac{\pi}{4}$.

9. Find the Laplace transform of $t \sin 2t$.

10. If $L\left[f(t)\right] = \frac{s}{(s+2)^3}$, find the value of $\lim_{t\to 0} f(t)$

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Solve the equation $(D^2 + 4D + 3)y = e^{-x} \sin x$. (6)

(ii) Solve the equation $(D^2 + 1)y = x \sin x$ by the method of variation of parameters. (10)

Or

(b) (i) Solve $(x+2)^2 \frac{d^2y}{dx^2} - (x+2)\frac{dy}{dx} + y = 3x + 4$. (8)

(ii) Solve $\frac{dx}{dt} + y = \sin 2t$

$-x + \frac{dy}{dt} = \cos 2t$. (8)

12. (a) (i) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$. (8)

(ii) Prove that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational vector and find the scalar potential such that $\vec{F} = \Delta\phi$. (8)

Or

(b) Verify Gauss divergence theorem for $\vec{F} = 4xz\,\vec{i} - y^2\,\vec{j} + yz\,\vec{k}$ over the cube bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1$. (16)

13. (a) (i) Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and determine its harmonic conjugate. (8)

(ii) Prove that the analytic function with constant modulus is also constant. (8)

Or

(b) (i) Find the image of $|z-3i|=3$ under the mapping $w=\frac{1}{z}$. (8)

(ii) Find the bilinear transformation which maps the points $z=0,-i,-1$ into $w=i,1,0$. (8)

14. (a) (i) Evaluate $\int_c \frac{z+4}{z^2+2z+5}\,dz$, where c is the circle $|z+1-i|=2$ using Cauchy's integral formula. (8)

(ii) Using residue theorem, evaluate $\int_c \frac{3z^2+z-1}{(z^2-1)(z-3)}\,dz$ where c is the circle $|z|=2$. (8)

Or

(b) (i) Find the Laurent's series of $f(z)=\frac{7z-2}{z(z+1)(z-2)}$ in $1<|z+1|<3$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$. (8)

15. (a) (i) Find the Laplace transform of $\frac{e^{-t}\sin 2t}{t}$. (8)

(ii) Find the Laplace transform of the square wave function defined by

$$f(t)=\begin{cases} 1; & 0<t<a \\ -1; & a<t<2a \end{cases}$$ (8)

Or

(b) (i) Using convolution theorem find the inverse Laplace transform of $\frac{1}{(s^2+1)(s+1)}$. (8)

(ii) Solve the initial value problem $y''-3y'+2y=4t,\ y(0)=1,\ y'(0)=-1$. (8)

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