Reg. No. : | | | | | | | | | | |

Question Paper Code: 41102

B.E. / B.Tech. DEGREE EXAMINATION, DECEMBER 2014.

First Semester

Civil Engineering

14UMA102 - ENGINEERING MATHEMATICS – I

(Common to all branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 1 = 10 Marks)

1. If the Eigen values of the matrix $A=\begin{bmatrix}1 & 1\\ 3 & -1\end{bmatrix}$ are 2, -2 then the Eigen values of A^T are

   (a) $\frac{1}{2}, \frac{-1}{2}$ (b) 2, -2 (c) 1, -1 (d) 1, 3

2. If two of the Eigen values of $\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$ are 2 and 8, then the third Eigen value is

   (a) -2 (b) 0 (c) 2 (d) 3

3. Examine the nature of the series $1 + 2 + 3 + 4 + \ldots\ldots\ldots + n + \ldots\ldots\infty$

   (a) divergent (b) convergent (c) oscillatory (d) linear

4. The geometric series $1 + r + r^2 + r^3 + \ldots\ldots + r^n + \ldots\ldots$ converges if

   (a) $r \leq 1$ (b) $r \geq 1$ (c) $r > 1$ (d) $r < 1$

5. What is the radius of curvature at (3, 4) on the curve $x^2 + y^2 = 25$?

(a) 5 (b) -5 (c) 25 (d) -25

6. The envelope of the family of straight lines $y = mx + \frac{1}{m}$, m being the parameter is

(a) $y^2 = -4x$ (b) $x^2 = 4y$ (c) $y^2 = 4x$ (d) $x^2 = -4y$

7. If $x = r\cos\theta$ and $y = r\sin\theta$ then $\frac{\partial r}{\partial x}$ is

(a) $\frac{y}{\sqrt{x^2+y^2}}$ (b) $\frac{x}{\sqrt{x^2+y^2}}$ (c) $\frac{y}{\sqrt{x^2-y^2}}$ (d) $\frac{x}{\sqrt{x^2-y^2}}$

8. If $u = \frac{y}{z} + \frac{z}{x}$ then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is

(a) u (b) –u (c) 2u (d) 0

9. After changing the order of integration in $\int_0^a \int_x^a f(x, y)\, dydx$, y varies from

(a) 0 to x (b) 0 to a (c) x to a (d) 0 to y

10. The value of the double integral $\int_0^\pi \int_0^a r\, dr\, d\theta$ is

(a) πa^2 (b) $\frac{\pi a^2}{2}$ (c) $\frac{\pi r^2}{2}$ (d) πr^2

PART - B (5 x 2 = 10 Marks)

11. Write down the quadratic form corresponding to the matrix $A = \begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$

12. Show that the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots$ is convergent.

13. Find the envelope of the family of circles $(x-\alpha)^2 + y^2 = r^2$, where α being the parameter.

14. If $x = u^2 - v^2$ and $y = 2uv$, find the Jacobian of x and y with respect to u and v.

15. Evaluate $\int_0^2 \int_0^\pi r\sin^2\theta\, d\theta\, dr$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Using Cayley-Hamilton theorem, find A^4 for $A=\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ (8)

(ii) Find the eigen values and eigen vectors of $A=\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ (8)

Or

(b) Reduce the quadratic form $6x^2+3y^2+3z^2-4xy-2yz+4zx$ into canonical form by an orthogonal transformation. (16)

17. (a) (i) Test the convergence of the series $\sum_{n=0}^{\infty} ne^{-n^2}$ (8)

(ii) Test for convergence of the series

$$1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+....+\frac{2^n-2}{2^n+1}x^{n-1}+....(x>0)$$ (8)

Or

(b) (i) Discuss the convergence of the series

$$\frac{x}{x+1}-\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}-.....(0<x<1)$$ (8)

(ii) Prove that the series $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+.....+(-1)^{n+1}\frac{1}{\sqrt{n}}+......$ is conditionally convergent. (8)

18. (a) (i) Find the radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$. (8)

(ii) Find the evolute of the parabola $y^2 = 4ax$. (8)

Or

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(b) (i) Find the envelope of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ where the parameters are related by the equation $a^2+b^2=c^2$ (8)

(ii) Considering the evolute as envelope of normals, find the evolute of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (8)

19. (a) (i) If $u=\frac{yz}{x}, v=\frac{zx}{y}, w=\frac{xy}{z}$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ (8)

(ii) Find the extreme values of the function $f(x, y) = x^3 + y^3 - 12x - 3y + 2$ (8)

Or

(b) (i) Expand $e^x \log(1+y)$ in powers of x and y upto terms of third degree. (8)

(ii) A rectangular box open at the top, is to have a volume of 32cc. Find the dimensions of the box that requires the least material for its construction. (8)

20. (a) (i) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$. (8)

(ii) Evaluate $\iiint \frac{dx\, dy\, dz}{\sqrt{1-x^2-y^2-z^2}}$ for all positive values of *x*, *y*, *z* for which the integral is real. (8)

Or

(b) (i) Find by double integration, the area between the two parabolas $y^2=9x$ and $x^2=9y$ (8)

(ii) By transforming into polar coordinates, evaluate $\iint \frac{x^2y^2}{x^2+y^2}dxdy$ over the annular region between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$, $b>a$. (8)