Reg. No. : | | | | | | | | | | |

Question Paper Code: 41261

M.E. DEGREE EXAMINATION, DECEMBER 2014.

First Semester

Structural Engineering

14PMA125 – APPLIED MATHEMATIS FOR STRUCTURAL ENGINEERING

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (5 x 1 = 5 Marks)

1. The value of $L[H(t-3)] =$

   (a) $\frac{e^{-3}}{s}$ (b) $\frac{3}{s^2}$ (c) $\frac{e^{-3s}}{s}$ (d) $\frac{1}{s^3}$

2. Newton-Raphson iterative formula for finding the square root of a positive real number N is

   (a) $x_{i+1} = \frac{3(x_i + N)}{2x_i}$ (b) $x_{i+1} = \frac{(x_i^2 + N)}{2x_i}$

   (c) $x_{i+1} = \frac{x_i - 2x_i}{x_i^2 + N}$ (d) None of the above

3. The functional $I\left[y(x)\right] = \int_0^1 \left(xy + y^2 - 2y^2 - 2y^2 y'\right) dx \quad y(0) = 1,\ y(1) = 2$ has

   (a) Unique extremal (b) No extremal

   (c) Variational problem is meaningless (d) None of the above

4. The technique effectively used to calculate smallest eigen value is

   (a) Inverse iteration (b) Forward iteration

   (c) Sweeping technique (d) None of these

5. The range of regression coefficient is

   (a) (-1,1) (b) (0,1) (c) $(-\infty,\infty)$ (d) (0,1)

PART - B (5 x 3 = 15 Marks)

6. Obtain Fourier transform of Dirac Delta function.

7. State distinct features of the Monte-Carlo Method.

8. Write down the Ostrogradsky equation for the functional

$$I(z) = \int_D \int \left[\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 - 2z \right] dx dy.$$

9. When can we expect faster convergence in power method?

10. Let X denote the proportion of allotted time that a randomly selected Student spends working on a certain aptitude test. Suppose the pdf of X is

$$f(x, \theta) = \begin{cases} (\theta + 1) x^{\theta} & 0 \le x \le 1 \\ 0 & otherwise \end{cases}.$$

Where -1 < θ. A random sample of ten students yields data

$$x_1 = 0.92, x_2 = 0.79, x_3 = 0.90, x_4 = 0.65, x_5 = 0.86,$$
$$x_6 = 0.47, x_7 = 0.73, x_8 = 0.97, x_9 = 0.94, x_{10} = 0.77$$

Use the method of moments to obtain an estimator of θ, and then compute the estimate for this data

PART - C (5 x 16 = 80 Marks)

11. (a) Using the Laplace transform method, solve the IBVP described as

$$PDE: u_{xx} = \frac{1}{c^2} u_{tt} - \cos \omega t, \quad 0 \le x < \infty, \quad 0 \le t < \infty$$
$$BCs: u(0,t) = 0, \quad u \text{ is bounded as } x \text{ tends to } \infty$$
$$ICs: u_t(x,0) = u(x,0) = 0$$

(16)

Or

(b) Solve the heat conduction problem described by

$$PDE: k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < \infty, \quad t > 0$$
$$BC: u(0,t) = u_{0,} \quad t \ge 0$$
$$IC: u(x,0) = 0, \quad 0 < x < \infty$$

u and $\partial u / \partial x$ both tend to zero as $x \to \infty$ (16)

12. (a) (i) Solve the equations $\begin{bmatrix} 4 & 1 & 1 \\ 1 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}$ using the method of cholesky. (8)

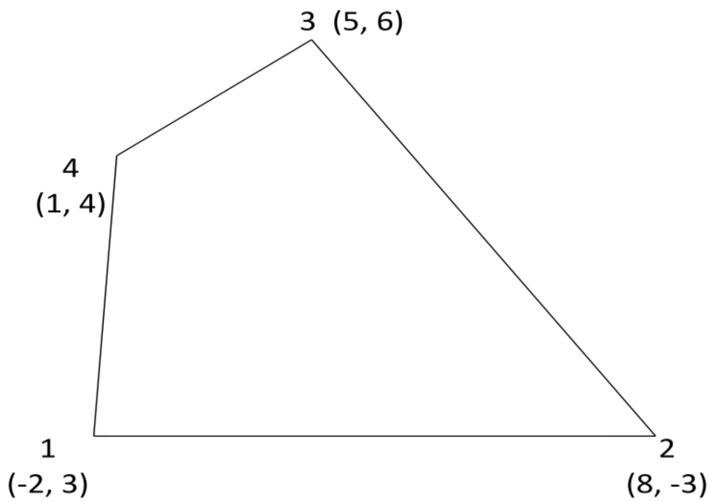
(ii) Use Gauss – Hermite formula to evaluat:

(a) $\int_{-\infty}^{\infty} \frac{e^{-x^2}}{1+x^2}\, dx$

(b) $\int_{-\infty}^{\infty} e^{-x^2}\, dx$ (8)

Or

(b) (i) Find the area of the quadrilateral shown in figure. Use mapping function for a quadrilateral (8)



(ii) Solve, by Gauss-Seidel iteration method

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 10$$
(8)

13. (a) Find an approximate solution to the problem of the extremum of the functional $V\left[y(x)\right] = \int_0^1 \left(y'^2 - y^2 + 2xy\right) dx, \quad y(0) = y(1) = 0$ by Rayleigh Ritz method and Compare it with the exact solution. (16)

Or

(b) (i) Find the curve with fixed boundary points such that its rotation about the axis of abscissa give rise to a surface of revolution of minimum surface area. (8)

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(ii) Determine the extremal of the functional

$$I\left[y(x)\right]=\int_{-l}^{l}\left(\frac{1}{2}\mu y''^{2}+\rho y\right)dx$$

Subject to $y(-l)=0,\ y'(-l)=0,\ y(l)=0,\ y'(l)=0$. (8)

14. (a) (i) Obtain the characteristic polynomial of the matrix using Faddeev-Leverrier method

$$A=\begin{bmatrix}3 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 3\end{bmatrix}$$ (8)

(ii) Determine the largest eigen value and the corresponding eigen vector to the matrix using the power method

$$A=\begin{bmatrix}4 & 1 & 0\\ 1 & 20 & 1\\ 0 & 1 & 4\end{bmatrix}$$ (8)

Or

(b) Explain Rayleigh -Ritz method for finding eigen values. Find the eigen value of

$$\begin{bmatrix}4 & -2 & 0\\ -2 & 8 & -2\\ 0 & -2 & 4\end{bmatrix}\{\phi\}=\lambda\begin{bmatrix}1 & & \\ & 2 & \\ & & 1\end{bmatrix}\{\phi\}$$ (16)

15. (a) (i) Let $X_1 \ldots, X_n$ be a random sample of size n from a normal distribution.Obtain the maximum likelihood estimators of μ *and* σ^2. Also obtain the maximum likelihood estimator σ. (8)

(ii) The following table gives age *(x)* in years of cars and annual maintenance cost *(y)* in hundred rupees:

X	1	3	5	7	9
Y	15	18	21	23	22

Estimate he maintenance cost for a 4 year old car after finding the appropriate line of regression. (8)

Or

(b) In a trivariate distribution:

$$\sigma_1=2, \sigma_2=\sigma_3=3, r_{12}=0.7, r_{23}=r_{31}=0.5$$

Find (i) $r_{23.1}$ (ii) $R_{1.23}$ (iii) $b_{12.3}$, $b_{13.2}$ and (iv) $\sigma_{1.23}$ (16)