Reg. No. : | | | | | | | | | | |

Question Paper Code: 41221

M.E. DEGREE EXAMINATION, DECEMBER 2014.

First Semester

Communication Systems

14PMA122 - APPLIED MATHEMATICS FOR COMMUNICATION ENGINEERS

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART - A (5 x 1 = 5 Marks)

1. $J_o'(x) =$ __________
   (a) $J_o^2(x)$ (b) $-J_1(x)$ (c) $J_o(x)$ (d) $J_1(x)$

2. If A is an orthogonal matrix, then A^{-1} is __________ .
   (a) Unitary (b) Orthogonal (c) Zero (d) None of these

3. The Partial differential equation $f_{xx} + 2f_{xy} + 4f_{yy}$ is known as __________ .
   (a) Parabolic (b) Hyperbolic (c) Elliptic (d) None of these

4. If the feasible region of a LPP is empty, then the solution is __________ .
   (a)Infeasible (b) Unbounded (c) Alternative (d) Basic feasible

5. For $(M/M/1):(\infty/FIFO)$ model, $L_q =$ ______ .
   (a) $\frac{\lambda}{\mu}$ (b) $1-\frac{\lambda}{\mu}$ (c) $\frac{\lambda}{\mu-\lambda}$ (d) $\frac{\lambda^2}{\mu(\mu-\lambda)}$

PART - B (5 x 3 = 15 Marks)

6. Prove that $J_{\frac{5}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\left\{\left(\frac{3-x^2}{x^2}\right)\sin x - \frac{3}{x}\cos x\right\}$.

7. Given the three vectors $\alpha = (2, 0, -1)$, $\beta = (0, -1, 0)$ and $\gamma = (2, 0, 4)$ in R^3 find $\|\alpha\|, \|\beta\|, \textit{ and } \|\gamma\|$.

8. Write down the different solutions of one dimensional wave equation.

9. Give the steps of MODI method in solving a transportation problem.

10. A hospital receives an average of 3 emergency calls in a 10 minute interval. What is the probability that there are at the most 3 emergency calls in a 10 minute interval?

PART - C (5 x 16 = 80 Marks)

11. (a) (i) State and prove orthogonal property of Bessel's function. (8)

(ii) Prove that $\frac{d}{dx}\left[x\, J_n(x)\, J_{n+1}(x)\right] = x\left[J^2{}_n(x) - J^2{}_{n+1}(x)\right]$. (8)

Or

(b) (i) Prove that $e^{\frac{1}{2}x\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$. (8)

(ii) Show that $J_0^{\,2} + 2J_1^{\,2} + 2J_2^{\,2} + 2J_3^{\,2} + ... = 1$. (8)

12. (a) (i) Find the Pseudo inverse of a matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$. (8)

(ii) Find out eigen values of $A = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$ by QR Algorithm. (8)

Or

(b) (i) Find the Canonical form of $A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$. (8)

(ii) Find the least square lines for the points (-1, 0), (1, 1), (2, 1), (3, 2) and (5, 3). (8)

13. (a) Using Laplace transformation method, solve the initial boundary value problem $u_{tt} = u_{xx}, \quad 0 < x < 1, \; t > 0$ with the boundary conditions $u(0,t) = 0 = u(1,t), \quad t > 0$ and the initial conditions $u(x,0) = \sin \pi\, x, \quad u_t(x,0) = -\sin \pi x, \; 0 < x < 1.$ (16)

Or

(b) A string is stretched and fixed between two points (0, 0) and $(l, 0)$. Motion is initiated by displacing the string in the form $u = \lambda \sin\left(\frac{\pi x}{l}\right)$ and released from rest at t = 0. Find the displacement of any point on the string at time t. (16)

14. (a) (i) I Use Simplex method to solve the LPP.

Minimize $Z = 8x_1 - 2x_2$

Subject to the constraints:

$-4x_1 + 2x_2 \le 1;$

$5x_1 - 4x_2 \le 3$ and

$x_1, x_2 \ge 0.$ (8)

(ii) Solve the transportation problem (8)

	1	2	3	4	Supply
	21	16	25	13	11
	17	18	14	23	13
	32	27	18	41	19
Demand	6	10	12	15	

Or

(b) (i) Use Two-Phase simplex method to solve

Maximize $Z = 5x_1 + 8x_2$

Subject to the constraints:

$3x_1 + 2x_2 \ge 1$

$x_1 + 4x_2 \ge 3$

$x_1 + x_2 \ge 5$ and

$x_1, x_2 \ge 0$ (8)

(ii) Five operators have to be assigned to five machines. The assignment costs are given in the table below:

Machines

Operators	**1**	**2**	**3**	**4**	**5**
A	5	5	-	2	6
B	7	4	2	3	4
C	9	3	5	-	3
D	7	2	6	7	2
E	6	5	7	9	1

Operator A cannot operate machine 3 and operator C cannot operate machine 4. Find the optimal assignment schedule. (8)

15. (a) (i) Customers arrive at a bank counter handled by a single person, according to a Poisson process with a mean rate of 10 per hour. The time required to serve a customer has an exponential distribution with a mean of 4 minutes. Find the following:

1. The average number of customers in the system
2. The average waiting time of a customer in the queue
3. The probability that there would be 2 customers in the queue. (8)

(ii) A Super market has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in a Poisson fashion at the rate of 10 per hour. Find the following:

1. What is the probability of having to wait for service?
2. What is L_q and L_s? (8)

Or

(b) (i) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour.

1. Find the effective arrival rate at the clinic.
2. What is the probability that an arriving patient does not have to wait?
3. What is the expected waiting time until a patient is discharged from the clinic? (8)

(ii) Self-service system is followed in a supermarket at Chennai. The customer arrivals occur according to a Poisson distribution with mean 40 per hour. Service time per customer is exponentially distributed with mean 6 minutes.

1. Find the expected number of customers in the system
2. What is the percentage of time that the facility is idle? (8)