Reg. No. : | | | | | | | | | | |

Question Paper Code: 31001

B.E. / B.Tech. DEGREE EXAMINATION, OCTOBER 2014.

Third Semester

Civil Engineering

01UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. State the conditions for $f(x)$ to have Fourier series expansion.

2. If $f(x) = \sin x, \quad in -\pi < x < \pi$, find the values of a_0 and a_0 ?

3. Define Fourier Integral theorem.

4. Find the Fourier sine transform of $\cos x, 0 < x < a$.

5. Find the z - transform of $\{n\}$.

6. State initial and final value theorems on z - transform.

7. Write down the three possible solutions of one dimensional heat equation.

8. A string is stretched and fastened to two points ℓ apart. Motion is started by displacing the string into the form $y = y_0 \sin\left(\frac{\pi x}{\ell}\right)$ from which it is released at time $t = 0$. Formulate this problem as the boundary value problem.

9. Write the diagonal five point formula to solve the equation $u_{xx} + u_{yy} = 0$.

10. State Crank – Nicholson scheme to solve $u_{xx} = a u_t$ when $k = ah^2$.

PART - B (5 x 16 = 80 Marks)

11. (a) Expand $f(x) = x^2$ when $-\pi \le x \le \pi$ in a Fourier series of periodicity 2π. Hence deduce that

(i) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \text{........} = \frac{\pi^2}{12}$

(ii) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \text{........} = \frac{\pi^2}{6}$. (16)

Or

(b) (i) Expand the function $f(x) = \sin x, 0 < x < \pi$ in a Fourier cosine series. (8)

(ii) Find a Fourier series up to two harmonic to represent $f(x)$ (8)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1	1.4	1.9	1.7	1.5	1.2	1

12. (a) (i) Show that the Fourier transform of $e^{-x^2/2}$ is self reciprocal. (8)

(ii) Find the Fourier transform of $f(x) = 1 - |x|$ if $|x| < 1$ and hence find the value of

$\int_0^\infty \frac{\sin^4 t}{t^4} dt.$ (8)

Or

(b) Find the Fourier cosine and sine transform of $e^{-ax}, a > 0$ and hence evaluate $\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$ and $\int_0^\infty \frac{x^2}{(a^2 + x^2)^2} dx$

(16)

13. (a) (i) Find the inverse z - transform of $\frac{z^2}{(z-a)(z-b)}$ using convolution theorem. (8)

(ii) Solve $y_{n+2}+6y_{n+1}+9y_n=2^n$ given $y_0=y_1=0$. (8)

Or

(b) (i) Find the inverse z - transform of $\frac{z}{(z-1)^2(z+1)}$. (8)

(ii) Solve $y_{n+2}-4y_{n+1}+4y_n=0$ given $y_0=1, y_1=0$. (8)

14. (a) A tightly stretched string with fixed end points $x=0$ *and* $x=\ell$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(\ell-x)$. Find the displacement. (16)

Or

(b) A rod of length *l* has its ends *A* and *B* maintained at 0^o C and 100^o C respectively, until steady state conditions prevail. If *B* is suddenly reduced to 0^o C and maintained at 0^oC, find the temperature at a distance *x* from *A* at time *t*. (16)

15. (a) Solve $\nabla^2 u=-10(x^2+y^2+10)$ over the square mesh with sides $x=0,\quad y=0,\quad x=3,\quad y=3$ with $u=0$ on the boundary and mesh length 1 unit. (16)

Or

(b) Solve numerically $4u_{xx}=u_{tt}$ with the boundary conditions $u(0,t)=0,\quad u(4,t)=0$ and the initial conditions $u_t(x,0)=0$ and $u(x,0)=x\quad(4-x)$ taking $h=1$ up to *4* time steps. (16)