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 **Reg. No. :**

**Question Paper Code: 11002**

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2014.

First Semester

Civil Engineering

01UMA102 ENGINEERING MATHEMATICS – I

 (Common to all branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. The product of two eigenvalues of the matrix $A=\left[\begin{matrix} 6&-2& 2\\-2& 3&-1\\ 2&-1& 3\end{matrix} \right]$ is 16. Find the third eigenvalue.

2. Determine the nature of Q.F. $f\left(x, y, z\right)=2xy+2yz+2zx$.

3. Show that the plane $2x-2y+z+12=0$ touches the sphere

 $x^{2}+y^{2}+z^{2}-2x-4y+2z-3=0$ .

4. Find the equation of the right circular cone with vertex at the origin, whose axis is

 $\frac{x}{1}=\frac{y}{-1}=\frac{z}{2}$ and with a semi-vertical angle $30^{°}$.

5. Find the radius of curvature of the curve $xy=c^{2}$ at $(c, c)$.

6. Find the envelope of $x\cos(θ+y\sin(θ=a))$ where $θ $ is the parameter.

7. If $u=sin^{-1}\left(\frac{x}{y}\right)+tan^{-1}\left(\frac{y}{x}\right) ,$ using Euler’s theorem show that $x\frac{∂u}{∂x}+y \frac{∂u}{∂y}=0$.

8. If $x=u^{2}-v^{2}$ and $y=2uv ,$ find $\frac{∂(x, y)}{∂(u, v)}$.

9. Evaluate  $r dθ dr$.

10. Sketch roughly the region of integration for $ f\left(x, y\right)dy dx$.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix $A=\left[\begin{matrix}2&1&1\\1&2&1\\0&0&1\end{matrix}\right]$ . (8)

 (ii) Using Cayley Hamilton theorem find the inverse of the matrix $ A=\left[\begin{matrix}1& 0& 3\\2& 1&-1\\1&-1& 1\end{matrix}\right]$ . (8)

Or

(b) Reduce the quadratic form $x^{2}+y^{2}+z^{2}-2xy-2yz-2zx$ to canonical form by orthogonal reduction. Also find the nature of the quadratic form. (16)

12. (a) (i) Find the equation of the sphere having the circle $x^{2}+y^{2}+z^{2}+10y-4z-8=0 ,$ $x+y+z=3$ as the greatest circle. (8)

 (ii) Find the equation of the cone formed by rotating the line $ 2x+3y=6 , z=0$
 about the $y-axis$. (8)

Or

(b) (i) Find the equation of the sphere which touches the sphere

$x^{2}+y^{2}+z^{2}+2x-6y+1=0 $ at the point $(1, 2, -2)$ and passes through the origin . (8)

 (ii) Find the equation to the right circular cylinder whose guiding circle is

 $x^{2}+y^{2}+z^{2}=9 , x-y+z=3$ . (8)

13. (a) (i) Find the envelope of the lines $\frac{x\cos(θ)}{a}+\frac{y\sin(θ)}{b}=1 , θ $being the parameter. (8)

(ii) Find the equation of evolute of the parabola $y^{2}=4ax$. (8)

Or

(b) (i) Find the circle of curvature of the curve $xy=12$ at $(3, 4)$. (8)

(ii) Find the evolute of the parabola $y^{2}=4x$ considering it as the envelope of its
normals. (8)

14. (a) (i) If $∅\left(x,y\right)=f(u,v) $ where $u=x^{2}-y^{2} and v=2xy$ prove that

 $\frac{∂^{2}f}{∂x^{2}}+\frac{∂^{2}f}{∂y^{2}}=4\left(x^{2}+y^{2}\right)\left(\frac{∂^{2}∅}{∂u^{2}}+\frac{∂^{2}∅}{∂v^{2}}\right)$ . (8)

 (ii) Find the minimum value of $x^{2}+y^{2}+z^{2}$ given that $x+y+z=3a$ . (8)

Or

(b) (i) Expand $x^{2}y+3y-2$ in powers of $(x-1)$ and $(y+2)$ upto third degree

terms using Taylor’s theorem. (8)

(ii) Examine $f\left(x,y\right)=x^{3}+y^{3}-12x-3y+20$ for its extreme values. (8)

15. (a) (i) Change the order of integration in  $\frac{x dx dy}{x^{2}+y^{2}}$ and hence evaluate it. (8)

 (ii) Evaluate  $\frac{x dy dx}{\sqrt{x^{2}+y^{2}}}$ by changing into polar co-ordinates. (8)

Or

 (b) (i) Evaluate  over the area bounded by the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$. (8)

(ii) Express the volume of sphere $x^{2}+y^{2}+z^{2}=a^{2}$ as a volume integral and hence evaluate it. (8)