Reg. No. :

# **Question Paper Code: 12001**

M.E. DEGREE EXAMINATION, MAY 2014.

#### First Semester

Computer Science and Engineering

#### 01PMA121 - OPTIMIZATION TECHNIQUES

[Common to Computer Science and Engineering (with Specialization in Networks)]

(Regulation 2013)

Duration: Three hours

Answer ALL Questions.

Maximum: 100 Marks

PART A - (10 x 2 = 20 Marks)

- 1. Sketch the block diagram of the basic structure of a queuing process.
- 2. What are the characteristics of a server in a queuing system?
- 3. State the Pollaczek-Khintchine formula.
- 4. What are the differences between open and closed queuing networks?
- 5. What are the characteristics of Monte- Carlo simulation?
- 6. What are the limitations of simulation on the applications in queuing theory?
- 7. How do you obtain the solution of the dual from the simplex table of primal?
- 8. Show that assignment problem is the special case of the transportation model.
- 9. What is Bordered Hessian Matrix? Give an example.
- 10. Write down the Kuhn-Tucker conditions for solving a non-linear programming problem.

### PART - B (5 x 16 = 80 Marks)

- 11. (a) Workers come to tool store room to inquire about special tools (required by them) for accomplishing a particular project assigned to them. The average time between two arrivals is 60 seconds and the arrivals are assumed to be in Poisson distribution. The average service time (of the tool room attendant) is 40 seconds. Determine
  - (i) Average queue length.
  - (ii) Average length of non-empty queues.
  - (iii) Average number of workers in system including the worker being attended.
  - (iv) Mean waiting time of an arrival.
  - (iv) Average waiting time of an arrival who waits.
  - (v) The type of policy to be established. In other words, determine whether to go in for the additional number of tool store room attendant which will minimize the combined cost of attendants' idle time and the cost of workers' waiting time. Assume the charges of a skilled worker Rs. 4 per hour and that of tool store room attendant Rs 0.75 per hour. (16)

## Or

- (b) (i) Ships arrive at a port at the rate of one in every 4 hours with exponential distribution of interarrival times. The time a ship occupies a berth for unloading has exponential distribution with an average of 10 hours. If the average delay of ships waiting for berths is to be kept below 14 hours, how many berths should be provided at the port?
  - (ii) A self-service store employs one cashier at its counter. Nine customers arrive on an average every 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate, find
    - (i) Average number of customers in the system.
    - (ii) Average number of customers in queue or average queue length.
    - (iii) Average time a customer spends in the system.
    - (iv) Average time a customer waits before being served. (8)

- 12. (a) A bank has two tellers working on saving accounts. The first teller handles withdrawals only while the second teller handles deposit only. It has been found that the service time distribution for the deposits and withdrawals both is exponential with mean service time 3 minutes per customer. Depositors are found to arrive in a Poisson fashion throughout the day with mean arrival rate 16 per hour. Withdrawers also arrive in a Poisson fashion with mean arrival rate of 14 per hour.
  - (i) What would be the effect on the average waiting time for depositors and withdrawers if each teller could handle both withdrawals and deposits? (10)
  - (ii) What would be the effect if this could only be accomplished by increasing the service time to 3.5 minutes? (6)

#### Or

- (b) (i) For  $\{(M/M/1) : (\alpha / FCFS)\}$  queuing model, derive Pollaczek-Khintchine formula. (8)
  - (ii) A firm is engaged in both shipping and receiving activities. The management is always interested in improving the efficiency of new innovations in loading and unloading procedures. The arrival distribution of trucks is found to be Poisson with arrival rate of 3 trucks per hour. The service time distribution is exponential with unloading rate of 4 trucks per hour. Determine
    - (i) The expected number of trucks in the queue
    - (ii) The expected waiting time of a truck in the queue
    - (iii) Probability that the loading and unloading docks and workers will be idle.(8)
- 13. (a) A sample of 100 arrivals of customers at a retail sales depot is according to the following distribution:

Time between arrivals	0.5	1	15	2	25	2	25	4	15	5
(minutes):	0.5	1	1.5	Ζ	2.5	3	5.5	4	4.5	3
Frequency:	2	6	10	25	20	14	10	7	4	2

A study of the time required to service customers by adding up the bills, receiving payments, placing packages, etc. yields the following distribution:

Service Time (minutes): 0.5 1 1.5 2 2.5 3

Frequency: 12 21 36 19 7 5

Estimate the average percentage customer waiting time and average percentage idle time of the server by simulation for the next 10 arrivals. (16)

#### Or

(b) The following table gives the arrival pattern at a coffee counter for 'one minute' intervals. The service is taken as 2 person in one minute in one counter.

No. of persons arriving:	0	1	2	3	4	5	6	7
Probability percentage:	5	10	15	30	20	10	5	5

Using Monte Carlo simulation technique and the following random numbers, generate the pattern of arrivals and the queue formed when the following 20 random numbers are given:

5, 25, 16, 80, 35, 48, 67, 79, 90, 92, 9, 14, 1, 55, 20, 71, 30, 42, 60 and 85.

Find the queue length if two counters are used i.e., 4 persons in one minute. (16)

14. (a) Use Simplex method to solve the following Linear Programming problem.

$$\begin{array}{lll} \mbox{Minimize} & Z = x_1 - 3x_2 + 3x_3 \\ \mbox{Subject to} & 3x_1 - x_2 + 2x_3 \leq 7 \\ & 2x_1 + 4x_2 \geq -12 \\ & -4x_1 + 3x_2 + 8x_3 \leq 10 \\ & \mbox{and } x_1, x_2, x_3 \geq 0 \end{array} \tag{16}$$

- Or
- (b) (i) Find the optimal transportation cost to the following transportation problem in which the cells contain the transportation cost in rupees. (8)

	$\mathbf{W}_1$	$W_2$	<b>W</b> <sub>3</sub>	$W_4$	<b>W</b> <sub>5</sub>	Available
F <sub>1</sub>	7	6	4	5	9	40
F <sub>2</sub>	8	5	6	7	8	30
F <sub>3</sub>	6	8	9	6	5	20
F <sub>4</sub>	5	7	7	8	6	10
Required	30	30	15	20	5	100 (Total)

(ii) A company has a team of four salesman and there are four districts where the company wants to start its business. After taking into account the capabilities of salesman and the nature of districts, the company estimates that the profit per day in rupees for each salesman in each district is as below.

Districts

		1	2	3	4
Salesman	А	16	10	14	11
	В	14	11	15	15
	С	15	15	13	12
	D	13	12	14	15

Find the assignment of salesman to various districts which will yield maximum profit.

15. (a) (i) Use the method of Lagrangian multipliers to solve the following non-linear programming problem.

Minimize 
$$Z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$$
  
subject to the constraints  $x_1 + x_2 + x_3 = 7$   
 $x_1, x_2, x_3 \ge 0$  (8)

(ii) Find the maximum of the function 
$$f(X) = 2x_1 + x_2 + 10$$
  
subject to  $g(X) = x_1 + 2x_2^2 = 3$  using the Lagrange multiplier method. (8)

(b) Use the Kuhn-Tucker conditions to solve the following non-linear programming problem.