Reg. No. : | | | | | | | | | | |

Question Paper Code: 54004

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2017

Fourth Semester

Electronics and Communication Engineering

15UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If λ is equal to 8 then standard deviation of exponential probability distribution is

   (a) 0.425 (b) 0.125 (c) 0.225 (d) 0.325

2. Formula of variance of uniform or rectangular distribution is as

   (a) $\frac{(b-a)^2}{6}$ (b) $\frac{(b-a)^3}{8}$ (c) $\frac{(b-a)^2}{12}$ (d) $\frac{(b+a)^2}{2^2}$

3. If X and Y are independent random variables with mean 2 & 3 and variances 1 & 2 respectively, then the mean of $Z = 2X - 5Y$ is

   (a) -11 (b) - 8 (c) -37 (d) 19

4. The value of correlation co-efficient lies in the interval

   (a) $(0, \infty)$ (b) $[0,1]$ (c) $(-1,1)$ (d) $[-1,1]$

5. Stochastic process are

   (a) Random in nature (b) are function of time
   (c) both of the mentioned (d) none of these

6. Arrivals at a telephone booth are considered to be Poisson with an average time of 12min. between one arrival and the next. The length of a phone call is assumed to be

distributed exponentially with mean 4 minutes, then the fraction of the day when the phone will be in use is

(a) 1/2 (b) 1/3 (c) 2/3 (d) 3/2

7. The power spectrum of a WSS process $X(t)$ is given by $S(\omega) = \frac{1}{\omega^2 + 4}$. then its average power is given by

(a) 0 (b) 1/2 (c) 1/4 (d) 1

8. The auto correlation function of stationary process $X(t)$ is given by $R_{XX}(\tau) = 25 + \frac{4}{1 + 6\tau^2}$ then the mean of the stationary process is.

(a) 29 (b) 5 (c) 25 (d) 25 + (4 / 7)

9. Convolution is used to find

(a) amount of similarity between the signals
(b) response of the system
(c) multiplication of the signals
(d) Fourier transform

10. Which one of the following systems are linear (i) $y(t) = t\,x(t)$ and (ii) $y(t) = x^2(t)$

(a) (i) only (b) (ii) only (c) both (d) none

PART - B (5 x 2 = 10 Marks)

11. A continuous random variable X has probability density function (pdf) $f(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$. Find k such that $P(X > k) = 0.5$.

12. If the joint pdf of (X, Y) is $f_{XY}(x, y) = \begin{cases} e^{-(x+y)} & x > 0, y > 0 \\ 0 & otherwise \end{cases}$, check whether X and Y are independent.

13. State the postulates of a Poisson process.

14. Find the variance of the stationary ergodic process $X(t)$ whose auto correlation function is given by $R_{XX}(\tau) = 36 + \frac{4}{1 + 3\tau^2}$.

15. D Define white noise process.

PART - C (5 x 16 = 80 Marks)

16. (a) A Random Variable X has the following probability distribution

$X=x$	0	1	2	3	4	5	6	7
$P[X=x]$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) the value of k (ii) $P\left[\frac{1.5<X<4.5}{X>2}\right]$ (iii) the smallest value of λ for which $P(X \le \lambda) > 1/2$ (16)

Or

(b) Obtain the moment generating function of geometric distribution. Hence, find its mean and variance. (16)

17. (a) The joint probability mass function of (X,Y) is given by $p(x,y)=k(2x+3y)$ for $x=0,1,2$; $y=1,2,3$. Find all marginal and conditional distributions. Also find the probability distribution of $X+Y$. Are X and Y independent. (16)

Or

(b) If X and Y are independent random variables with pdf's $e^{-x}, x \ge 0$ and $e^{-y}, y \ge 0$ respectively, find the density function of $U=\frac{X}{X+Y}$ and $V=X+Y$. Are U and V independent? (16)

18. (a) A man drives a car or catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tosses a fair die and drove to work if and only if a 6 appeared. Find (i) the probability that he takes a train on third day (ii) probability that he drives to work in the long run. (16)

Or

(b) (i) Prove that the difference of two independent Poisson processes is not a Poisson process. (8)

(ii) Given a random variables Y with characteristic function $\varphi(\omega)=E[e^{i\omega Y}]$ and a random process $X(t)=\cos(\lambda t+Y)$. Show that $X(t)$ is WSS if $\varphi(1)=0, \varphi(2)=0$. (8)

19. (a) State and Prove the Wiener–Khinchine theorem. (16)

Or

(b) The auto correlation function of the random binary transmission $X(t)$ is given by $R(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T} & |\tau| < T \\ 0 & |\tau| \geq T \end{cases}$. Find the power spectrum of the process. (16)

20. (a) A circuit has an impulse response given by $h(t) = \begin{cases} 1/T & 0 \leq t \leq T \\ 0 & otherwise \end{cases}$. Evaluate $S_{YY}(\omega)$ in terms of $S_{XX}(\omega)$ (16)

Or

(b) Consider a system with transfer function $\dfrac{1}{1 + jw}$. An input signal with autocorrelation function $m\,\delta(\tau) + m^2$ is fed as input to the system. Find the mean and the mean-square value of the output. (16)