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**Question Paper Code: 44004**

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2017

Fourth Semester

Electronics and Communication Engineering

14UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2014)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

(Smith chart may be permitted)

PART A - (10 x 1 = 10 Marks)

- The mean value of Poisson distribution is  
(a)  $\theta$                       (b) 1                      (c) 0                      (d)  $\lambda$
- If  $X$  is a Random Variable, where  $\text{Var}(x) = 4$ , then predict the  $\text{Var}(3X+8)$   
(a) 36                      (b) 0                      (c) 26                      (d) 44
- When will the two Regression Lines be coincide  
(a)  $r=0$                       (b)  $r=1$                       (c)  $r = \pm 1$                       (d)  $r = \infty$
- If  $X=Y$  then correlation coefficient between them is  
(a) 0                      (b)  $\infty$                       (c) 1                      (d)  $\pm 1$
- A Non-Null Persistent and Aperiodic state is called  
(a) Return state                      (b) Irreducible                      (c) Ergodic                      (d) Recurrent
- Every Strongly stationary process of order 2 is a  
(a) Orthogonal process                      (b) Stationary Process  
(c) WSS Process                      (d) None of these

7. The power spectral density of  $X(t)$  is defined by

$$(a) y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$$

$$(b) X(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$$

$$(c) s_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)e^{i\omega\tau}d\tau$$

$$(d) s_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)e^{-i\omega\tau}d\tau$$

8.  $R_{XX}(\tau)$  is an \_\_\_\_\_ function of  $\tau$

(a) Positive

(b) 1

(c) Even

(d) Odd

9. White noise is also called as

(a) System

(b) White Gaussian noise

(c) Functional white noise

(d) Stationary

10. A \_\_\_\_\_ is a functional relationship between the input  $X(t)$  and the output  $Y(t)$

(a) system

(b) process

(c) functional

(d) stationary

PART - B (5 x 2 = 10 Marks)

11. State Axioms of Probability.

12. Define covariance.

13. Define random telegraph signal process.

14. State Wiener-Khinchine theorem.

15. If  $X(t)$  is a WSS process and if  $Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$ , then formulate

$$R_{XY}(\tau) = R_{XY}(\tau) * h(\tau).$$

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Deduce the moment generating function of exponential distribution and hence find its mean and variance. (8)

(ii) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are taken at random from this set, interpret the probability that at least one of them would have scored above 75? (Given the area between  $z=0$  and  $z=2$  under the standard normal curve is 0.4772). (8)

Or

(b) A random variable  $X$  has the following probability function

Value of $x$	0	1	2	3	4
$P(x)$	$k$	$3k$	$5k$	$7k$	$9k$

Find the value of  $k$ ,  $P(x < 3)$  and distribution function of  $x$ . (16)

17. (a) The joint probability distribution of  $X$  and  $Y$  is given below:

$Y \backslash X$	-1	1
0	1/8	3/8
1	2/8	2/8

Find the correlation coefficient between  $X$  and  $Y$ . (16)

Or

(b) (i) If the joint probability density function of a two dimensional random variable

$$(X, Y) \text{ is given by } f(x, y) = xy^2 + \frac{x^2}{8}, 0 \leq x \leq 2, 0 \leq y \leq 1.$$

Find out (i)  $P(X > 1)$ , (ii)  $P(Y < 1/2)$ . (8)

(ii) Analyse the correlation coefficient between the heights (in inches) of fathers  $X$  and their sons  $Y$  from the following data. (8)

$X$	65	66	67	67	68	69	70	72
$Y$	67	68	65	68	72	72	69	71

18. (a) Generalize the postulates of a Poisson process and derive the probability distribution for the Poisson Process. Also Show that the sum of two independent Poisson process is again a Poisson process. (16)

Or

(b) (i) Explain the classification of random process. (8)

(ii) The transition probability of a Markov chain  $\{X\}$ ,  $n = 1, 2, 3, \dots$ , having 3

states 1, 2 and 3  $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$  and the initial distribution is

$$p^{(0)} = (0.7, 0.2, 0.1).$$

Find (1)  $P\{X_2 = 3\}$  and (2)  $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$ . (8)

19. (a) (i) State and Prove Wiener-Khinchine theorem. (8)

(ii) Given that the autocorrelation function of a stationary random process is

$$R_{xx}(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}. \text{ Predict the mean and variance of the process } \{X(t)\}. \quad (8)$$

Or

(b) (i) Define cross-correlation function and write the properties of cross-correlation function. (8)

(ii) State and Prove Wiener-Khinchine theorem. (8)

20. (a) (i) The input to a time- invariant, stable linear system is a WSS process, Enumerate that the output will also be a WSS process. (8)

(ii) If  $X(t)$  is a band limited process such that  $S_{xx}(\omega) = 0, |\omega| > \sigma$ , then formulate  $2[R_{xx}(0) - R_{xx}(\tau)] \leq \sigma^2 \tau^2 R_{xx}(0)$ . (8)

Or

(b) (i) Show that  $S_{yy}(\omega) = S_{xx}(\omega)|H(\omega)|^2$  where  $S_{xx}(\omega)$  and  $S_{yy}(\omega)$  are the power spectral density functions of the input  $X(t)$ , output  $Y(t)$  and  $H(\omega)$  is the system transfer function. (8)

(ii) If the input to a time- invariant, stable linear system is a WSS process, Enumerate that the output will also be a WSS process. (8)