Reg. No. : | | | | | | | | | | |

Question Paper Code: 50022

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2017

Second Semester

Civil Engineering

15UMA202 – ENGINEERING MATHEMATICS - II

(Common to ALL Branches)

(Regulation 2015)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The general solution of the linear differential equation $a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_n y = X$ is

   (a) $y =$ Complementary Funtion (b) $y =$ Particular Integral

   (c) $y = C.F + P.I$ (d) $y = C.F * P.I$

2. If n is a positive integer, then $\frac{1}{f(D)}\ x^n$ is equivalent to

   (a) $\frac{1}{f(-D^2)}\ x^n$ (b) $\frac{1}{f(n)}\ x^n$ (c) $\frac{1}{f(n)}\ x^{-n}$ (d) $[f(D)]^{-1}\ x^n$

3. Del operator is

   (a) same as the gradient operator (b) vector differential operator

   (c) both (a) and (b) (d) none of these

4. Vector field $\vec{P}$ is irrotational if $\nabla \times \vec{P} =$

   (a) ∞ (b) -1 (c) 1 (d) 0

5. A single valued function w = f(z) of a complex variable z is said to be analytic at a point Z_o if it has

(a) second derivative at Z_o (b) a unique derivative at Z_o
(c) second derivative at Z (d) a unique derivative at Z

6. Which of the following is not true when f(z) = u+iv is analytic at a point

(i) $u_x = v_y$ at the point (ii) $u_y = -v_x$ at the point (iii) $u_{xx}+u_{yy} = 0$ at the point
(iv) u_x, u_y, v_x, v_y are continuous at the point

(a) only (i) (b) only (i) and (ii)
(c) all are false (d) all are true

7. Cauchy's integral theorem is also known as

(a) Integral formula (b) Cauchy's theorem
(c) C-R equation (d) None of these

8. In Taylor's series taking a=0, the series is reduces to

(a) Fourier series (b) Maclaurin's series
(c) Laurent's series (d) None of these

9. $L\left(t^3\right)$ is equal to

(a) $L(f(t)) = \frac{1}{1-e^{ST}}\int_0^T e^{st}f(t)dt$ (b) $L(f(t)) = \frac{1}{1-e^{ST}}\int_0^T e^{-st}f(t)dt$
(c) $L(f(t)) = \frac{1}{1-e^{-ST}}\int_0^T e^{-st}f(t)dt$ (d) $L(f(t)) = \frac{1}{1-e^{-ST}}\int_0^T e^{st}f(t)dt$

10. The $L^{-1}\left(\frac{1}{s}\right)$ is

(a) 1 (b) 2 (c) 3 (d) 0

PART - B (5 x 2 = 10 Marks)

11. Find the complementary function of $(D^2 - 4D + 3)y = 2\,e^x$.

12. State Stoke's theorem.

13. Define bilinear transformation.

14. State Cauchy' integral theorem.

15. State initial and final value theorem.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Solve the equation $(D^2 + 6D + 9)y = e^x + sin3x$. (8)

(ii) Solve the equation $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 32(logx)^2$. (8)

Or

(b) (i) Solve $\frac{d^2y}{dx^2} + 4y = 4tan2x$ using method of variation of parameter. (8)

(ii) Solve $(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$. (8)

17. (a) (i) Find the directional derivative of $\phi = 4xz^2 + x^2yz - 3z$ at the point $(1, -2, -1)$ in the direction $2\vec{i} - \vec{j} - 2\vec{k}$. (8)

(ii) Show that:
$\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential. (8)

Or

(b) Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ over the cub bounded by $x = 0, x = a, y = 0, y = b, z = 0$ and $z = c$. (16)

18. (a) (i) Find the analytic function $w = u + iv$ if $v = e^{2x}(xcos2y - ysin2y)$. Hence find u. (8)

(ii) If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (8)

Or

(b) (i) Show that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic and determine its conjugate. (8)

(ii) Find the bilinear transformation that maps the points $z_1 = 1,\ z_2 = -1,\ z_3 = 1$ into the $w_1 = 0,\ w_2 = 1,\ w_3 = \infty$ respectively. (8)

19. (a) (i) Use Cauchy's integral formula to evaluate $\int_c \frac{sin\pi z^2 + cos\pi z^2}{(z-1)(z-3)} dz$, where c is the circle $|z| = 4$. (8)

(ii) Expand $f(z) = \frac{(z^2-1)}{(z+2)(z+3)}$ in a Laurent's series if $(i)|z| > 3,\ (ii)\ 2 < |z| < 3.$ (8)

Or

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5sin\theta}$ using contour integration. (16)

20. (a) (i) Find the laplace transform of the rectangular wave given by

$$f(t) = \begin{cases} 1, & 0 < t < b \\ -1. & b < t < 2b \end{cases} \quad \text{with } f(t+2b) = f(t).$$ (8)

(ii) Verify initial and final value theorem for $f(t) = (t^2 + 4t + 4)e^{-t}$. (8)

Or

(b) (i) Use convolution theorem to find $L^{-1}\left[\frac{1}{(s+1)(s+2)}\right]$. (8)

(ii) Using Laplace transform method to solve:

$y'' - 4y' + 8y = e^{2t},\ y(0) = 2,\ y'(0) = -2.$ (8)