Reg. No. : | | | | | | | | | | |

Question Paper Code: 50044

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2017

Fourth Semester

Electronics and Communication Engineering

15UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2015)

Duration: Three hours　　　　Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. What is the value of k if $f(x) = kx^2, 0 < x < 3$.

   (a) 1/9　　(b) 1/8　　(c) 1/7　　(d) 1/6

2. Six coins are tossed 6,400 times. What is the probability of getting 6 holds, *x* times?

   (a) e^{-100x}　　(b) $\frac{1}{x}\ e^{-100x}$　　(c) $\frac{1}{x!}\ e^{-100}\ \ 100^{x}$　　(d) None of these

3. $F_{XY}\ (\infty, \infty)$ is equal to

   (a) 0　　(b) 1　　(c) ∞　　(d) None of these

4. $E(XY) =$ _________ if *X* and *Y* are independent.

   (a) *XE(Y)*　　(b) *YE(X)*　　(c) *E(X) E(Y)*　　(d) 0

5. If *T* is discrete and *S* is continuous, the random process is called as

   (a) Discrete random sequence　　(b) Continuous random sequence

   (c) Discrete random process　　(d) Continuous random process

6. State i is said to be periodic with period $d_i\ if\ d_i\ =$

   (a) 1　　(b) 0　　(c) <1　　(d) >1

7. If the processes $X(t) and\ Y(t)$ are orthogonal ,then $R(\tau) =$

(a) 1 (b) 0 (c) -1 (d) None

8. what is the mean square value of the process $X(t)$, if its ACF is $R(\tau) = e^{-(\tau^2}/2)$.

(a) 2 (b) 3 (c) 1 (d) 5

9. A zero-mean white Gaussian noise is passed through an ideal low pass filter of bandwidth 10 *kHz*. The output is uniformly sampled with sampling period $t_S = 0.03$ *msec*. The samples so obtained would be

(a) correlated (b) statistically independent
(c) uncorrelated (d) orthogonal

10. The probability density function of the envelope of narrow band Gaussian noise is

(a) Poisson (b) Gaussian (c) Rayleigh (d) Rician

PART - B (5 x 2 = 10 Marks)

11. State Baye's theorem.

12. Find the value of *K*, if $f\ (x, y) = K\ (1 - x)(1 - y)$, for $0 < x,\ y < 1$. Is to be a joint density function.

13. Define Wide sense stationary process.

14. Find the power spectral density of a WSS process with autocorrelation function of $R(\tau) = e^{-\alpha\tau^2}$.

15. Describe a linear system.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) A continuous random variable *X* has a PDF $f\ (x) = 3x^2, 0\ \leq x\ \leq 1$. Find the value of *K* and α such that (1) $P\ (X\ \leq K) = P\ (\ X\ > K)$ (2) $P\ (\ X\ > \propto) = 0.1$ (8)

(ii) Find the MGF, mean and variance of the Binomial distribution. (8)

Or

(b) (i) There are 3 true coins and 1 false coin with head on both sides. A coin is chosen at random and tossed 4 times. If head occurs all the 4 times, what is the probability that the false coin has been chosen and used? (8)

(ii) The mileage which car owners get with a certain kind of radial tyre is a random variable having an exponential distribution with mean 40,000 *km*. Find the probabilities that one of these tires will last (1) at least 20,000 *km* and (2) at most 30,000 *km*. (8)

17. (a) (i) The joint pdf of random variable *(X,Y)* is given by $f(x,y) = kxye^{-(x^2+y^2)}, x > 0, y > 0.$ Find the value of k and also prove that X and Y are independent. (8)

(ii) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of $U = X - Y$. (8)

Or

(b) (i) The joint probability mass function of *(X,Y)* is given by $p(x,y) = k(2x+3y),\ x = 0,1,2;\ y = 1,2,3.$ Find the value of k and all the marginal probability distribution function. Also find the probability distribution of $(X + Y)$. (8)

(ii) If X, Y and Z are uncorrelated random variables with zero means and standard deviations 5, 12 and 9 respectively and if $U = X + Y$ and $V = Y + Z$, find the correlation coefficient between U and V. (8)

18. (a) (i) Show that the random process $X(t) = Acos(wt + \theta)$ is wide-sense stationary, if A and w are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) Customers arrive at a one man barber shop according to a Poisson process with a mean inter arrival time of 12 *min*. Customers spend an average of 10 *min* in the barber's chair.

(1) what is the expected number of customers in the barber shop and in the queue?

(2) How much time can a customer expect to spend in the barber's shop and queue? (8)

Or

(b) (i) Prove that the difference of two independent Poisson processes is not a Poisson process. (8)

(ii) A fair die is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X_n\}$. Find P^2 *and* $P(X_2 = 6)$. (8)

19. (a) State and Prove the Wiener–Khinchine theorem. (16)

Or

(b) (i) A random process {X(t)} is given by $X(t) = A\cos pt + Bsinpt$, where A and B are independent random variables such that $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = \sigma^2$. Find the power spectral density of the process. (8)

(ii) If the power spectral density of a WSS process is given by $S(\omega) = \begin{cases} \frac{b}{a}(a - |\omega|), |\omega| \leq a \\ 0, otherwise \end{cases}$, find the autocorrelation function of the process. (8)

20. (a) (i) A random process $\{X(t)\}$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t},\ t > 0$. If the autocorrelation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$, find the power spectral density of the output process $Y(t)$. (8)

(ii) Suppose $X(t)$ be the input process to a linear system with autocorrelation is $R_{XX}(\tau) = 3\delta(\tau)$ and the impulse response $H(\omega) = \frac{1}{6+i\omega}$ then find the autocorrelation of the output process $Y(t)$. (8)

Or

(b) (i) A system has an impulse response $h(t) = e^{\beta t}U(t)$, find the power spectral density of the output Y(t) corresponding to the input $X(t)$. (8)

(ii) Prove that the mean of the output of a linear system is given by $\mu_Y = H(0)\mu_X$, where $X(t)$ is WSS. (8)