Reg. No. : | | | | | | | | | | |

Question Paper Code: 50032

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2017

Third Semester

Civil Engineering

15UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to EEE, ECE, EIE, Mechanical and Chemical Engineering Branches)

(Regulation 2015)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If $f(x) = \begin{cases} cosx, 0 < x < \pi \\ 50, \pi < x < 2\pi \end{cases}$, and $f(x) = f(x + 2\pi)$ for all x, then the sum of the Fourier series of $f(x)$ at $x = \pi$ is

   (a) 50 (b) 51/2 (c) 49/2 (d) 51

2. The value of b_n for $xsinx$ in $(-\pi, \pi)$ is

   (a) π (b) $-\pi$ (c) 0 (d) $\frac{\pi}{2}$

3. If $f(x) = 1, -a < x < a$, then $F[f(x)]$ is

   (a) $\sqrt{\frac{2}{\pi}} \frac{sinas}{a}$ (b) $\sqrt{\frac{\pi}{2}}\left(\frac{sinas}{s}\right)$ (c) $\sqrt{\frac{2}{\pi}}\left(\frac{sinas}{a}\right)$ (d) $\sqrt{\frac{\pi}{2}}\left(\frac{sinas}{s}\right)$

4. The Fourier sine transform of $f(x) = e^{-ax}$ is

   (a) $\sqrt{\frac{2}{\pi}} \left(\frac{s^2}{s^2+a^2}\right)$ (b) $\frac{1}{\sqrt{2\pi}} \left(\frac{s}{s^2+a^2}\right)$ (c) $\frac{\sqrt{\pi}}{s} \left(\frac{s}{s^2+a^2}\right)$ (d) $\frac{1}{\sqrt{2}}\left(\frac{s}{s^2+a^2}\right)$

5. $Z[e^{-at}t] =$

   (a) $\frac{e^{aT}}{(z-e^{aT})^2}$ (b) $\frac{ze^{aT}}{(z-e^{aT})^2}$ (c) $\frac{Tze^{aT}}{(ze^{aT}-1)^2}$ (d) $\frac{Te^{aT}}{(ze^{aT}-1)^2}$

6. $Z\left[\frac{a^n}{n!}\right] =$

(a) e^z (b) $e^{a/z}$ (c) $e^{-a/z}$ (d) $e^{-z/a}$

7. The partial differential equation obtained from $z = (x^2 + a)(y^2 + b)$ is

(a) $pq = z$ (b) $pq = xy$ (c) $pq = 4xy$ (d) $4xyz = pq$

8. The complete integral of $p - q = 0$ is

(a) $z = ax + by + c$ (b) $z = ax + ay + c$
(c) $z = bx + ay + c$ (d) $z = pq$

9. The suitable solution of one dimensional heat equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ is

(a) $u(x,t) = (Acospx + Bsinpx)e^{-\alpha^2 p^2 t}$
(b) $u(x,t) = (Ae^{px} + Be^{-px})e^{-\alpha^2 p^2 t}$
(c) $u(x,t) = (Ax + B)e^{-px}$
(d) $u(x,t) = (Ax + B)e^{-\alpha^2 p^2 t}$

10. In ______ state, temperature do not depend on time ‘t’.

(a) steady (b) transient (c) absolute (d) bounded

PART - B (5 x 2 = 10 Marks)

11. Find the half range sine series of $f(x) = 2$ in $0 < x < \pi$.

12. Prove that $F[x^n f(x)] = (-i)^n \frac{d^n F(s)}{ds^n}$

13. State and prove initial value theorem.

14. Form the PDE by eliminating f from $z = xy + f(x^2 + y^2 + z^2)$.

15. The ends A and B of a rod of length 10cm have their temperature kept at 20℃ and 70℃. Find the steady state temperature distribution on the rod.

PART - C (5 x 16 = 80 Marks)

16. (a) Expand $f(x) = x(2\pi - x)$ as Fourier series in $(0, 2\pi)$and hence deduce that the sum of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$. (16)

Or

(b) Find the Fourier series of period 2π for the function $f(x) = x^2 + x$ in $-\pi < x < \pi$. Hence deduce $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$, assuming that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. (8)

17. (a) Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$. Hence deduce that $\int_0^{\infty} \frac{\sin t - t\cos t}{t^3} dt = \frac{\pi}{4}$. Using Parseval's identity show that $\int_0^{\infty} \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt = \frac{\pi}{15}$. (16)

Or

(b) (i) Show that $e^{-x^2/2}$ is self-reciprocal under Fourier Cosine transform . (8)

(ii) Evaluate $\int_0^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$ by using transform methods. (8)

18. (a) (i) Find $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$ (8)

(ii) Find $Z(r^n \cos n\theta)$ and $Z(r^n \sin n\theta)$. (8)

Or

(b) (i) Find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z-1)}$ by using convolution theorem. (8)

(ii) Using Z-transform, solve $y_{n+2} - 5y_{n+1} + 6y_n = 4^n$, given that $y_0 = 0, y_1 = 1$. (8)

19. (a) (i) Solve $z = px + qy + p^2 - q^2$. (8)

(ii) Solve $(D^2 - 5DD' + 6D'^2)z = \cos(x + 2y) + y\sin x$. (8)

Or

(b) (i) Solve $x(y^2 + z)p + y(x^2 + z)q = z(x^2 - y^2)$. (8)

(ii) Solve $(D^2 - D'^2 - 3D + 3D')z = e^{x+2y} + xy$. (8)

20. (a) A string is stretched between two fixed points $x = 0$ and $x = 2l$ and is released from rest from the initial position is given by

$$y(x,0) = f(x) = \begin{cases} \frac{bx}{l}, 0 < x < l \\ \frac{-b}{l}(x - 2l), l < x < 2l \end{cases}.$$ Find the displacement of the string.

(16)

Or

(b) A rectangular plate with insulated surface s 10cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of the short edge $y = 0$ is given by

$u = \begin{cases} 20x, 0 \leq x \leq 5 \\ 20(10 - x), 5 \leq x \leq 10 \end{cases}$ and the two long edges $x = 0$ and $x = 10$ as well as the other short edge are at 0℃. Find the temperature $u(x, y)$ at any point (x, y) of the plate. (16)