Reg. No. : | | | | | | | | | | |

Question Paper Code: 31044

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2017

Fourth Semester

Electronics and Communication Engineering

01UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

(Statistical tables may be permitted)

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Let X be a continuous random variable with pdf $f(x) = \begin{cases} 3x & 0 < x < 1 \\ 0 & otherwise \end{cases}$, find $P(X \leq 0.6)$.

2. The mean and variance of the binomial distribution are 4 and 3 respectively. Find $(X \leq 1)$.

3. If $Y = -2X + 3$, find the $Cov(X, Y)$.

4. State the equations of the two regression lines. What is the angle between them?

5. Define wide sense stationary process.

6. State any two properties of Poisson process.

7. Define: Power spectrum.

8. Prove that $R_{XY}(\tau) = R_{YX}(-\tau)$..

9. Define a system. When is it call linear system?

10. State casual system.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Find moment generating function of gamma distribution and hence find its mean and variance. (8)

(ii) If X is uniformly distributed in (-2,2), find $P(X < 0)$ and

$P\left(|X - 1| \geq \frac{1}{2}\right)$ (8)

Or

(b) (i) A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white? (8)

(ii) State and prove the memory less property of geometric distribution. (8)

12. (a) Two random variables X and Y have the joint density function $f(x,y) = \begin{cases} 2 - x - y & 0 < x < 1, 0 < y < 1 \\ 0 & otherwise. \end{cases}$.
Find the correlation coefficient of (X , Y). (16)

Or

(b) (i) The joint probability density function of the two dimensional random variable is $f(x,y) = \begin{cases} \frac{8}{9}xy & 1 < x < y < 2 \\ 0 & otherwise \end{cases}$. Find (a) marginal density functions of X and Y (b) conditional density function of Y given X. (8)

(ii) The joint pdf of X and Y is given by $f(x,y) = e^{-(x+y)}, x > 0, y > 0$. Find the probability density function of $U = \frac{X+Y}{2}$. (8)

13. (a) (i) Show that the random process $X(t) = A\text{Cos}(\omega_0 t + \theta)$ is w.s.s. if A and ω are constants and θ is uniformly distributed random variables in (0, 2π). (8)

(ii) If the WSS process $\{X(t)\}$ is given by $X(t)=10\cos(100t+\theta)$, where θ is uniformly distributed over $(-\pi,\pi)$, prove that $\{X(t)\}$ is correlation ergodic. (8)

Or

(b) (i) Prove that the random process {X(t)} and {Y(t)} are defined by $X(t) = A\cos\omega_0 t + B\sin\omega_0 t, Y(t) = B\cos\omega_0 t - A\sin\omega_0 t$ are jointly wide-sense stationary, if A and B are uncorrelated zero mean random variables with the same variance. (8)

(ii) Find the mean and autocorrelation of the Poisson process. (8)

14. (a) State and Prove Wiener-Khintchine theorem, and hence find the power Spectral density of a WSS process *X(t)* which has an autocorrelation

$$R_{xx}(\tau) = A_0\left[1 - \frac{|\tau|}{T}\right], \quad -T \leq \tau \leq T. \quad (16)$$

Or

(b) (i) If {X(t)} is a WSS process with auto correlation function $R_{XX}(\tau)$ and if Y(t) = X(t+a)-X(t-a), show that $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau + 2a) - R_{XX}(\tau - 2a)$. (8)

(ii) Find the power spectral density of a WSS process with autocorrelation function $R(\tau) = e^{-\alpha\tau^2}$. (8)

15. (a) (i) A WSS process X(t) with $R_{XX}(\tau) = Ae^{-a|\tau|}$, where A and 'a' are real positive constants is applied to the input of an LTI systems with $h(t) = e^{-bt}.u(t)$, where b is a real positive constant. Find the power spectral density of the output of the system. (8)

(ii) X(t) is the input voltage to a circuit (system) and Y(t) is the output voltage. {X(t)} is a stationary random processes with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-a|\tau|}$. Find $\mu_Y, S_{yy}(\omega)$, if the power transfer function is $H(\omega) = \frac{R}{R+iL\omega}$. (8)

Or

(b) (i) Show that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$ where $S_{XX}(\omega)$ *and* $S_{YY}(\omega)$ are the power spectral density functions of the input *X(t)* and the output *Y(t)* and $H(\omega)$ is the system transfer function. (8)

(ii) The input to the RC filter is a white noise process with ACF $R_{XX}(\tau) = \frac{N_0}{2}\delta(\tau)$. If the frequency response $H(\omega) = \frac{1}{1 + j\omega RC}$, find the autocorrelation and the mean-square value of the output process Y(t). (8)
