Reg. No. : | | | | | | | | | | |

Question Paper Code: 41031

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2017

Third Semester

Civil Engineering

14UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Find the RMS value of $f(x) = x^2$

   (a) π (b) 2π (c) $\frac{\pi^2}{\sqrt{5}}$ (d) $\frac{\pi}{5}$

2. R.M.S value of f(x) = x in (-1,1) is

   (a) 0 (b) 1 (c) $\frac{1}{3}$ (d) $\sqrt{\frac{1}{3}}$

3. Find the Fourier sine transform of e^{-3x}

   (a) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+9}$ (b) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+9}$ (c) $\frac{a}{s^2+9}$ (d) $\frac{s}{s^2+9}$

4. Find the Fourier transform of $f(x) = \begin{cases} 1, |x| \le 1 \\ 0, |x| > 1 \end{cases}$

   (a) $\sqrt{\frac{2}{\pi}}\left(\frac{\sin s}{s}\right)$ (b) $\frac{\sin a}{s}$ (c) $\frac{\sin s}{s}$ (d) $\sqrt{\frac{2}{\pi}}\frac{\cos s}{s}$

5. $\lim_{z \to 1}$ (z-1) F(z) =

   (a) f(1) (b) $F(\infty)$ (c) $f(\infty)$ (d) f(0)

6. Evaluate $Z^{-1}\left[\frac{z}{(z-1)^2}\right]$

(a) 1 (b) n (c) n^2 (d) n^3

7. In one dimensional heat equation $u_t = \alpha^2 u_{xx}$. What is α^2?

(a) Velocity (b) Speed (c) Diffusivity (d) Displacement

8. A rod of length 40 cm whose one end is kept at 20°C and the other end is kept at 60°C is maintained so until steady state prevails. Find the steady state temperature at a location 15cm from A?

(a) 5 (b) 10 (c) 13 (d) 15

9. The finite difference approximation to $y_i' =$

(a) $\frac{y_{i+1} - y_{i-1}}{h}$ (b) $\frac{y_{i+1} + y_{i-1}}{h}$ (c) $\frac{y_{i+1} - y_{i-1}}{2h}$ (d) $\frac{y_{i+1} + y_{i-1}}{2h}$

10. Classify the partial differential equation $u_{xx} - 2u_{xy} + u_{yy} = 0, x, y > 0.$

(a) Parabolic (b) Elliptic (c) Hyperbolic (d) None of these

PART - B (5 x 2 = 10 Marks)

11. Find the constant a_0 of the Fourier series for the function f(x) = k , $0 \le x \le 2\pi$.

12. Write the Fourier Cosine transform pair.

13. State initial and final value theorem of Z – transform.

14. Define steady state condition on heat flow.

15. Classify the following equation: $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0.$

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Find the Fourier series for f(x) = x^2 in $-\pi \le x \le \pi$ and deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{6}$. (8)

(ii) Find the first two harmonic of the Fourier series of f (x). Given by (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f (x)	1	1.4	1.9	1.7	1.5	1.2	1.0

Or

(b) (i) Find the cosine series for f (x) = x in (0, π) and then using Parseval's theorem, show that $\frac{1}{1^4}+\frac{1}{3^4}+\ldots.=\frac{\pi^4}{96}$. (8)

(ii) Find the complex form of Fourier series of $f(x)$ if $f(x)=\sin ax \ in \ -\pi<x<\pi$. (8)

17. (a) (i) Find the Fourier transform of $f(x)=\begin{cases}1, & |x|\le a\\ 0, & |x|>a\end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin s}{s}ds$ and using Parseval's identity prove that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ (8)

(ii) Find the Fourier transform of $e^{-a^2x^2}$ Hence prove that $e^{\frac{-x^2}{2}}$ is self reciprocal with respect to Fourier transforms. (8)

Or

(b) Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0, & |x|>1\end{cases}$ .Hence deduce the value of

(i) $\int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}ds$ (ii) $\int_0^\infty \left[\frac{\sin s - s\cos s}{s^3}\right]^2 ds$ (16)

18. (a) (i) Find $Z(cosn\theta)$ and hence deduce $Z\left(\frac{cosn\pi}{2}\right)$ (8)

(ii) Using convolution theorem, find the inverse Z- transforms of $\frac{z^2}{(z+a)^2}$. (8)

Or

(b) (i) Solve $y_{n+2}+4y_{n+1}+3y_n=3^n$ with $y_0=0$and$y_1=1$. (8)

(ii) Find the inverse Z – transform of $\frac{z(z+1)}{(z-1)^3}$ by residue method. (8)

19. (a) A tightly stretched flexible string has its ends fixed at x = 0 and x = l. At time t = 0, the string is given a shape defined by f(x) = $k(lx-x^2)$where 'k' is a constant, and then released from rest. Find the displacement of any point x of the string at any time t > 0. (16)

Or

(b) A plate is in the form of the semi- infinite strip $0 \le x \le 10$, $0 \le y \le \infty$, whose surfaces is insulated. If the temperature at short edge $y = 0$ is given by $u = \begin{cases} 20x, & 0 \le x \le 5 \\ 20(10-x), & 5 \le x \le 10 \end{cases}$ and all the other three edges are kept at $0^0 C$. Find the steady state temperature at any point of the plate. (16)

20. (a) Solve the Poisson's equations $\nabla^2 u = -81xy$, $0 < x < 1$, $0 < y < 1$, h=1/3, *u(0,y)= u(x,0), u(1,y)=u(x,1)*=100. (16)

Or

(b) (i) Using Bender-Schmidt's method solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given $u(0,t) = 0, u(1,t) = 0$ and $u(x,0) = sin\pi x, 0 < x < 1$ and $h = 0.2$. Find the value of u upto t=0.1. (8)

(ii) Solve $y'' - y = 0$ with the boundary condition y(0)=0 and y(1)=1. (8)