Reg. No. : | | | | | | | | | | |

Question Paper Code: 31031

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2017

Third Semester

Civil Engineering

01UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL Branches)

(Regulation 2013)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. State Parseval's theorem in Fourier series.
2. Find the constant term in the Fourier series corresponding to $f(x) = \sqrt{1 - \cos x}$ expressed in the interval $(-\pi, \pi)$.
3. Find Fourier Sine Transform of $\frac{1}{x}$.
4. If F(s) is the Fourier transform of f(x), then find the Fourier transform of f(x-a).
5. Find the Z-transform of a^n.
6. Find $Z[\frac{1}{n(n+1)}]$.
7. Find the type of the partial differential equation $4u_{xx} + 4u_{xy} + u_{yy} + 2u_x - u_y = 0$.
8. State any two laws which are assumed to derive one dimensional heat equation.
9. Write down the implicit formula to solve one dimensional heat flow equation.
10. State Liebmann's iteration process formula.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Obtain the Fourier series for the function $f(x) = 1 + x + x^2$ in the interval $-\pi < x < \pi$ and hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^3} + \cdots = \frac{\pi^2}{6}$ (8)

(ii) Find the half range sine series for the function $f(x) = e^{-x},\ x > 0$. (8)

Or

(b) (i) Find the Half range cosine series for $y = x$ in $(0, l)$ and hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots \infty$. (8)

(ii) Find the Fourier series as the second Harmonic to represent the function of the following data (8)

x	0	1	2	3	4	5
y	9	18	24	28	26	20

12. (a) (i) Find the Fourier cosine transform of e^{-4x} and hence find the values of $\int_0^\infty \frac{\cos 2x}{x^2+16} dx$ and $\int_0^\infty \frac{x \sin 2x}{x^2+16} dx$ (8)

(ii) Find the Fourier cosine transform of $f(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & \pi < x < \infty \end{cases}$ (8)

Or

(b) (i) Find the Fourier transform of $e^{-a|x|}$ $if\ a > 0$ (8)

(ii) Evaluate using transform method $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$ (8)

13. (a) (i) Using Convolution theorem, evaluate $Z^{-1}\left[\frac{9z^3}{(3z-1)^2(z-2)}\right]$ (8)

(ii) Using Z-transform solve the equation $u_{n+2} - 5u_{n+1} + 6u_n = 4^n, u(0) = 0$ and $u(1) = 1$. (8)

Or

(b) (i) State and prove initial and final value theorem on Z- transform. (8)

(ii) Find $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$ by using method of Partial fraction. (8)

14. (a) A tightly stretched string of length l with fixed ends is initially in its equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3\left(\pi \frac{x}{l}\right)$. Find the displacement $y(x,t)$. (16)

Or

(b) The ends A and B of a rod l cm long have the temperature at $40^0 c$ and $90^0 c$ until steady state prevails. The temperature of the ends are then changed to $90^0 c$ and $40^0 c$ respectively. Find the temperature distribution in the rod at any time. (16)

15. (a) Solve $\nabla^2 u = -10\left(x^2 + y^2 + 10\right)$ over the square mesh with sides $x = 0$, $y = 0$, $x = 3$, $y = 3$ with $u = 0$ on the boundary and mesh length 1 unit. (16)

Or

(b) (i) Solve by Crank-Nicholson method, the equation $u_{xx} = u_t$ subject to $u(x,0) = 0, u(0,t) = 0$ and $u(1,t) = t$, for two time steps, taking $h = \frac{1}{2}$ and $k = \frac{1}{8}$. (8)

(ii) Solve $u_{xx} = 2u_t$ given

$u(0,t) = u(4,t) = 0. u(x,0) = x(4-x)$ taking $h = k = 1$.

Find the value of u upto t =5 using Bender-Schmidt explicit difference scheme. (8)
