Reg. No. : | | | | | | | | | | |

Question Paper Code: 51022

M.E. DEGREE EXAMINATION, MAY 2018

First Semester

Communication Systems

15PMA122 - APPLIED MATHEMATICS FOR COMMUNICATION ENGINEERS

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (5 x 1 = 5 Marks)

1. For the Bessel function $J_{\frac{1}{2}}(x)$ is equals to

   (a) $\sqrt{\frac{2}{\pi x}} \tan x$ (b) $\sqrt{\frac{2}{\pi x}} \sin x$ (c) $\sqrt{\frac{2}{\pi x}} \cos x$ (d) $\sqrt{\frac{2}{\pi x}} \cot x$

2. For an $(M / M / 1) : (\infty / FIFO)$ queue, average number of customers in the queue is

   (a) $\frac{\lambda}{\mu(\mu - \lambda)}$ (b) $\frac{\lambda^2}{\mu(\mu - \lambda)}$ (c) $\frac{\mu^2}{\lambda(\mu - \lambda)}$ (d) $\frac{\mu}{\lambda(\mu - \lambda)}$

3. The value of $L^{-1}\left[\frac{1}{s-2} + \frac{1}{s+2}\right]$ is

   (a) $2 \sin 2t$ (b) $2 \cos 2t$ (c) $2 \sinh 2t$ (d) $2 \cosh 2t$

4. The total number of allotment in assignment problem for m destination are

   (a) $m - 1$ (b) $2m - 1$ (c) $m - n - 1$ (d) $m - n + 1$

5. If $X = (1, -3, -2)^T$ and $Y = (1, m, -4)^T$ are orthogonal then the value of m is

(a) -1 (b) 0 (c) 3 (d) -3

PART - B (5 x 3 = 15 Marks)

6. Show that $J_0''(x) = \frac{1}{2}[J_2(x) - J_0(x)]$.

7. Define Toeplitz matrix.

8. Find the Laplace transform of $\sin^2 t$.

9. Define Slack and Surplus variables in the linear programming problem.

10. At a railway station, only one train is handled at a time. The railway yard sufficient only for two trains to wait while the other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them at an average of 6 per hour. Find the average number of trains in the systems.

PART - C (5 x 16 = 80 Marks)

11. (a) (i) State and prove orthogonal property of Legendre's polynomials. (10)

(ii) Show that $J_4(x) = \left(\frac{48}{x^3} - \frac{3}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x)$. (6)

Or

(b) (i) Prove that $J_0^2(x) + 2\left[J_1^2(x) + J_2^2(x) + J_3^2(x) + \dots\dots\dots\right] = 1$. (8)

(ii) Show that $P_5'(x) = 9P_4(x) + 5P_2(x) + P_0(x)$. (8)

12. (a) Solve the equations by least square method

$$x_1 + 2x_2 + 3x_3 + x_5 = 1$$

$$-x_1 + 2x_3 - 2x_4 + 3x_5 = 1$$ (16)

Or

(b) Find all the Eigen values of $A = \begin{pmatrix} 6 & 4 & 1 & 1 \\ 4 & 6 & 1 & 1 \\ 1 & 1 & 5 & 2 \\ 1 & 1 & 2 & 5 \end{pmatrix}$ using QR-factorization. (16)

13. (a) An infinitely long string having one end at $x = 0$ is initially at rest on the x-axis. The end $x = 0$ undergoes aperiodic transverse displacement described by $A_0 \sin \omega t,\ t > 0$. Find the displacement of any point on the string at any time t. (16)

Or

(b) Using Laplace transform, solve

PDE: $2u_t = u_{xx},\ 0 < x < l \qquad t > 0$

BCs: $u(0, t) = 0,\ u(l, t) = g(t) \qquad t > 0$

IC: $u(x, 0) = 0 \qquad 0 < x < l.$ (16)

14. (a) Use simplex method to solve the following LPP

Maximize $z = 4x_1 + 10x_2$ subject to the constraints:

$2x_1 + x_2 \le 50,\quad 2x_1 + 5x_2 \le 100,\quad 2x_1 + 3x_2 \le 100 : \quad x_1 \ge 0 \text{ and } x_2 \ge 0.$ (16)

Or

(b) Given $x_{13} = 50$ units, $x_{14} = 20$ units, $x_{21} = 55$ units, $x_{31} = 30$ units, $x_{32} = 35$ units, $x_{34} = 25$ units. Is it an optimal solution to the transportation problem:

$$\text{Available units} \begin{bmatrix} 6 & 1 & 9 & 3 \\ 11 & 5 & 2 & 8 \\ 10 & 12 & 4 & 7 \end{bmatrix} \begin{matrix} 70 \\ 55 \\ 90 \end{matrix}$$

Required units: 85 35 50 45

If not, modify it to obtain a better feasible solution. (16)

15. (a) (i) Assume that the goods trains are coming in a yard at the rate of 30 trains per day and suppose that the inter-arrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit 9 trains at a time, calculate the probability that the yard is empty and find the average queue length. (8)

(ii) A road transport company has one reservation clerk on duty at a time. He handles information of bus schedules and makes reservations. Customers arrive at a rate of 8 per hour and the clerk can service 12 customers on an average per hour, what is the average number of customers waiting for the service of the clerk and the average time a customer has to wait before getting service. (8)

Or

(b) (i) A petrol pump station has two pumps. The service times follow the exponential distribution with a mean of 4 minutes and cars arrive for service in a Poisson process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service. What proportion of time the pumps remain idle. (8)

(ii) A barber shop has two barbers and three chairs for waiting customers. Assume that customers arrive in a Poisson process at a rate of 5 per hour and that each barber services customers according to an exponential distribution with mean of 15 minutes. Further, if customer arrives and there are no empty chairs in the shop he will leave. Find the steady-state probabilities. What is the probability that the shop is empty? What is the expected number of customers in the shop? (8)