A **Reg. No. :** | | | | | | | | | | |

Question Paper Code: 52024

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2018

Fourth Semester

Electronics and Communication Engineering

15UMA424 - PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2015)

(Statistical tables may be permitted)

Duration: Three hours | Maximum: 100 Marks

PART A - (10 x 1 = 10 Marks)

1. When X and Y are independent random variables, P(A∩B)= CO1-App

(a) P(A) / P(B) (b) P(A) + P(B) (c) P(A) - P(B) (d) P(A) x P(B)

2. If X is a uniformly distributed random variable in the interval a < x <b, then its mean is CO1-R

(a) $\frac{a+b}{2}$ (b) $\frac{(b-a)}{12}$ (c) $\frac{a-b}{2}$ (d) $\frac{ab}{2}$

3. If the joint probability density function of a bivariate random variable (X,Y) is f(x,y) = k , 0 < x< 1, 0 < y < 1, then the value of k is CO2-App

(a) 2 (b)3 (c)1 (d)4

4. The regression coefficient b_{XY} = 0.2337 and b_{YX} = 0.6643 , then the correlation coefficient is CO2-R

(a) 0.498 (b) 0.394 (c) 0.239 (d) 0.25

5. Differences of two independent Poisson processes is CO3-R

(a) not Poisson process (b) a Gaussian process

(c) a Binomial process (d) a sine wave process

6. The random process that is not stationary in any sense is called CO3-R

(a) strongly stationary process (b) evolutionary process

(c) uniform process (d) Weak process

7. Auto correlation function is maximum at $\tau =$ CO4-R

(a) 0 (b) 2 (c) 1 (d) ∞

8. The spectral density function of a real random process is CO4-R

(a) an odd function (b) neither odd nor even function

(c) an even function (d) straight line function

9. The convolution form of the output Y(t) of a linear time invariant system with the input X(t) and the system weighting function h(t) CO5-R

(a) $\int_{-\infty}^{\infty} h(u)\ du$ (b) $\int_{-\infty}^{\infty} h(u)\, X(t-u)\, du$

(c) $\int_{-\infty}^{\infty} h(u)\, y(t-u)\, du$ (d) $\int_{-\infty}^{\infty} X(t-u)\, du$

10. If the power spectral density of white noise is $\frac{N_0}{2}$, then its auto correlation function CO5-R

(a) $\frac{N_0}{2}\delta(\tau)$ (b) $2\,\delta(\tau)$ (c) $\frac{1}{2}\delta(\tau)$ (d) $\delta(\tau)$

PART – B (5 x 2= 10Marks)

11. Define Binomial distribution. CO1-R

12. The following table gives the joint probability distribution of X and Y. CO2-E

X \ Y	1	2	3
1	0.1	0.1	0.2
2	0.2	0.3	0.1

Compute the marginal probability mass functions of X & Y.

13. Define Wide sense stationary process. CO3-R

14. Find Mean of the random process X(t) , CO4-E

where $R(\tau) = 16 + \frac{9}{1+16\tau^2}$

15. Define a linear time invariant system. CO5-R

PART – C (5 x 16= 80Marks)

16. (a) (i) A random variable 'X' has the following probability function CO1-App (8)

Values of X	-2	-1	0	1	2	3
Probability P[X = x]	0.1	K	0.2	2K	0.3	3K

1) Determine the value of 'K'.

2) Find $P[X < 2]$ and $P[-2 < X < 2]$.

3) Find the cumulative distribution function of X.

(ii) The distribution function of the random variable X is given by $F(x) = 1 - (1 + x)e^{-x}, x \geq 0$. Find the density function, mean and variance of X. CO1-App (8)

Or

(b) (i) Derive the moment generating function of Poisson distribution and hence obtain its mean and variance. CO1-App (8)

(ii) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. What is the probability that a student scores above 75? CO1-App (8)

17. (a) (i) The two dimensional random variable (X,Y) has the joint density function CO2-App (8)

$$f(x,y) = (2x + y)k \quad , \qquad x = 0,1,2 \;\; ; y = 0,1,2$$

(1) Find the value of k.

(2) Find the marginal distribution of X and Y.

(3) Find the conditional distribution of Y for X=x.

(ii) The joint probability density function of random variable X and Y is given by CO2-App (8)

$$f(x,y) = 4xye^{-(x^2+y^2)}, \;\; x > 0 \;\; , y > 0.$$

Check whether X and Y are independent or not..

Or

(b) (i) If the joint probability density function of (X,Y) is given by $f(x,y) = x + y\, ; 0 \le x \le 1; 0 \le y \le 1$, find the probability density function of U = XY. CO2-Ana (8)

(ii) The regression lines for two random variables X and Y are 8X – 10Y + 66 = 0 and 40X – 18Y = 214. Find CO2-Ana (8)

(a) the mean values of X and Y

(b) the correlation coefficient between X and Y.

18. (a) (i) Consider the random process X(t) = Ycos ω t, t$\geq$ 0, Where ω is a constant and Y is a uniform random variable over (0,1). Find the auto correlation function R(t, τ) of X(t) . CO3-Ana (8)

(ii) The probability distribution of the process {X(t)} is given by CO3-Ana (8)

$$P[X(t) = n] = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,3,\ldots\ldots\ldots\ldots \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$$

Show that it is not stationary.

Or

(b)	(i) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (1) more than 1 minute (2) between 1 minute and 2 minutes and (3) 4 minutes or less.	CO3-Ana	(8)
	(ii) Consider the process X(t) = A cos ω t + B sin ω t, where A and B are uncorrelated random variables each with mean 0 and variance 1 and ω is a positive constant. Show that the process X(t) is covariancestationary.	CO3-Ana	(8)

19.	(a)	(i) If $R_{XX}(\tau) = a^2 e^{-2\lambda\|\tau\|}$ is the autocorrelation function of a random telegraph signal process X(t) , obtain the power spectral density of X (t).	CO4-App	(8)
		(ii) If $Y(t) = X(t+a) - X(t-a)$, prove that $R_{yy}(\tau) = 2R_{xx}(\tau) - R_{xx}(\tau + 2a) - R_{xx}(\tau - 2a)$	CO4-App	(8)

Or

(b)	(i) The power spectral density of the random process is given by $S_{XX}(\omega) = \begin{cases} S_0 & -a < \omega < a \\ 0 & otherwise \end{cases}$. Find the autocorrelation function and also the mean square value.	CO4-App	(8)
	(ii) The cross-power spectrum of real random processes {X(t)} and {Yt)} is given by $S_{XY}(\omega) = \begin{cases} a + bj\omega & -1 < \omega < 1 \\ 0 & otherwise \end{cases}$ Find the cross-correlation function .	CO4-App	(8)

20.	(a)	A random process X(t) is the input to a linear system whose impulse function is $h(t) = 2e^{-t}; t \geq 0$. The autocorrelation of the function of the process is $R_{XX}(\tau) = e^{-2\|\tau\|}$.Find the power spectral density of the output process Y(t).	CO5-App	(16)

Or

(b) If X (t) is a WSS process and H(ω) is the system transfer function if CO5-App (16)

$Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$ then prove that

(i) $R_{XY}(\tau) = R_{XX}(\tau) * h(\tau)$

(ii) $R_{YY}(\tau) = R_{XY}(\tau) * h(-\tau)$

(iii) $S_{XY}(\omega) = S_{XX}(\omega) * H(\omega)$

(iv) $S_{YY}(\omega) = S_{XX}(\omega) * |H(\omega)|^2$