A **Reg. No. :** | | | | | | | | | | |

Question Paper Code: 53021

B.E./B.Tech. DEGREE EXAMINATION, MAY 2018

Third Semester

Civil Engineering

15UMA321 Transforms and Partial Differential Equations

(Common to Mechanical, ECE, EEE, Chemical, AGRI, BME)

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer All Questions

PART A - (10x 1 = 10 Marks)

1. In the expansion of $xcosx$ as a Fourier series in *(-l, l)* the value of $a_n =$ CO1- R

   (a) 1 (b) -1 (c) l (d) 0

2. In the Fourier series of x^2 in the interval $(-l, l)$ CO1- R

   (a) $a_0 = 0$ but $a_n \neq 0$ (b) $b_n = 0$ (c) $a_0 = 0$ and $a_n = 0$ (d) none

3. If F(s) = F{f(x)},then F{f(x−a)} = CO2- R

   (a) e^{-isx} F(s) (b) s + a (c) s ─ a (d) e^{isa} F(s)

4. If F{f(x)} = f(s),then f(x) is said to be CO2- R

   (a) self reciprocal (b) multi reciprocal (c) mono reciprocal (d) nano reciprocal

5. The Z- transform of (3.4^n) = CO3- R

   (a) $\frac{3z}{z-4}$ (b) $\frac{3z}{z+4}$ (c) $\frac{z}{z-4}$ (d) $\frac{3}{z-4}$

6. If $Z[\{f(n)\}] = F(Z)$ then $Z[\{a^n\ f(n)\}]$ CO3- R

(a) $F(\frac{z}{a})$ (b) $F(az)$ (c) $F(a^n z)$ (d) $F(\frac{z}{a^n})$

7. The complete integral of p – q = 0 is given by CO4- R

(a) a = b + 2 (b) a = b + 1 (c) a = b (d) a = b − 1

8. A solution that contains as many arbitrary constants as there are independent variables is called as CO4- R

(a) singular integral (b) general integral (c) complete integral (d) particular integral

9. How many conditions needed to solve a one dimensional heat equation? CO5- R

(a) 3 (b) 2 (c) 4 (d) 5

10 In one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, what is α^2 CO5- R

(a) $\frac{Tension}{Mass}$ (b) Gravity (c) Friction (d) $\frac{Mass}{Friction}$

PART – B (5 x 2= 10Marks)

11. Find the half range sine series of $f(x) = 2$ in $0 < x < \pi$. CO1- U

12. Find the Fourier sine transform of $\frac{1}{x}, 0 < x < \infty$. CO2- U

13. Solve $Z[na^n]$. CO3- App

14. Form the PDE by eliminating f from $z = xy + f(x^2 + y^2 + z^2)$ CO4- R

15. Write all possible solutions for one dimensional wave equation. CO5- U

PART – C (5 x 16= 80Marks)

16. (a) (i)Find the fourier series expansion $f(x) = \begin{cases} x, & 0 < x < \pi \\ 2\pi - x, & \pi < x < 2\pi \end{cases}$ CO1-App (8) and hence deduce that

. $\frac{1}{1^2} + \frac{1}{2^2} + \cdots = \frac{\pi^2}{8}$

(ii) Find the complex form of Fourier series for the function $f(x)=e^{-x}$ in $-1< x < 1$. CO1-App (8)

Or

(b) (i) Find the Fourier series of $y = x^2$ in $-\pi < x < \pi$ and show that $1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$. CO1 -App (8)

(ii) Calculate the first two harmonics of the Fourier series from the following table: CO1 -App (8)

x	0	π/3	2π/3	π	4π/3	5π/3	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

17. (a) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & |x|<1 \\ 0, & |x|>1 \end{cases}$. CO2 -App (16)

And hence ,

Evaluate (i) $\int_0^{\infty} \frac{\sin t - t\cos t}{t^3} dt$ (ii) $\int_0^{\infty} \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt$

Or

(b) Find the sine and cosine transform of e^{-ax}, $a > 0$. Hence, Evaluate CO2 -App (16)

$\int_0^{\infty} \frac{x^2}{(x^2+a^2)^2} dx$ and $\int_0^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$.

18. (a) (i) Find $Z\left(\frac{1}{n(n-1)}\right)$ CO3- App (8)

(ii) Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2n$ given $y_0 = y_1 = 0$, using Z-transform. CO3- App (8)

Or

(b) (i) Find the inverse Z- transform of $\frac{8z^2}{(2z-1)(4z-1)}$ by using convolution theorem. CO3- App (8)

19. (a) Solve $(D^3 - 7DD'^2 - 6D'^3) z = e^{2x+y} + \sin(x + 2y) + x^2y$. CO4-App (16)

Or

(b) (i) Solve $z = px + qy + \sqrt{p^2 + q^2 + 1}$. CO4 -App (8)

(ii) Solve $x(y - z) p + y(z - x) q = z(x - y)$. CO4 -App (8)

20. (a) A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{\ell}\right)$. It is released from rest from this position. Find the displacement at any time't'. CO5- App (16)

Or

(b) An infinitely long plate of width $\pi\ cms$ with insulated surfaces has its temperature $u = 0$ on both long sides and one of the shorter sides. The temperature along the short edge $y = 0$ is given by $u(x, 0) = 3x, 0 < x < \pi$. Find the steady state temperature distribution $u(x, y)$. CO5- App (16)