Reg. No. : | | | | | | | | | | |

Question Paper Code: 43021

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2018

Third Semester

Civil Engineering

14UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The formula for finding the Euler constant a_n of a Fourier series in [0,2 π] is ___

(a) $a_n = \int_0^{\pi} f(x) cosnx\, dx$ (b) $a_n = \int_0^{2\pi} f(x) cosnx\, dx$

(c) $a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) cosnx\, dx$ (d) $a_n = \frac{1}{\pi} \int_0^{\pi} f(x) cos\frac{n\pi x}{l}\, dx$

2. The complex form of Fourier series of *f(x)* in $(-\ell, \ell)$ is given by, *f(x)* = $\sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{l}}$, where C_n is,

(a) $\frac{2}{l}\int_{-l}^{l} f(x) e^{-\frac{in\pi x}{l}} dx$ (b) $\frac{1}{2l}\int_{-l}^{l} f(x) e^{-\frac{in\pi x}{l}} dx$

(c) $\frac{2}{l}\int_{0}^{l} f(x) e^{-\frac{in\pi x}{l}} dx$ (d) $\frac{1}{2l}\int_{0}^{l} f(x) e^{\frac{in\pi x}{l}} dx$

3. $F\left(e^{-|x|}\right)$

(a) does not exist (b) $\frac{2is}{(s^2+1)}$ (c) $\frac{1}{\sqrt{2\pi}}\frac{s}{(s^2+1)}$ (d) $\frac{1}{\sqrt{2\pi}}\frac{2}{(s^2+1)}$

4. Fourier sine transform of *xf(x)* is,

(a) $F_c'(s)$ (b) $F_s'(s)$ (c) - $F_c'(s)$ (d) - $F_s'(s)$

5. $Z(n(n-1))$

(a) $\frac{2z}{(z+1)^3}$ (b) $\frac{z(z-1)}{(z-1)^3}$ (c) $\frac{2z}{(z-1)^3}$ (d) $\frac{z}{(z-1)^2}$

6. $Z(\cos^2 t)$

(a) $\frac{z}{2(z-1)}+\frac{z(z-\cos 2T)}{2(z^2-2z\cos 2T+1)}$ (b) $\frac{z}{(z-1)}+\frac{z(z-\cos 2T)}{(z^2-2z\cos 2T+1)}$

(c) $\frac{z}{2(z-1)}-\frac{z(z-\cos 2T)}{2(z^2-2z\cos 2T+1)}$ (d) $\frac{z}{(z-1)}-\frac{(z-\cos 2T)}{(z^2-2z\cos 2T+1)}$

7. When the ends of a rod is non zero for one dimensional heat flow equation, the temperature function *u(x,t)* is modified as the sum of steady state and transient state temperatures. The transient part of the solution which,

(a) increases with increase of time (b) decreases with increase of time
(c) increases with decrease of time (d) increases with decrease of time

8. The two dimensional heat flow equation in steady state is,

(a) $\frac{\partial^2 u}{\partial t^2}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ (b) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

(c) $\frac{\partial u}{\partial t}=\alpha^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$ (d) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

9. In the explicit formula for solving one dimensional heat equation, λ is ____

(a) $\frac{a}{h^2}$ (b) ah (c) $\frac{k}{ah^2}$ (d) ka

10. The standard five point formula in solving Laplace equation over a region is,

(a) $u_{ij}=\frac{1}{4}\left(u_{i-1,j+1}+u_{i-1,j}+u_{i+1,j}+u_{i+1,j-1}\right)$ (b) $u_{ij}=\frac{1}{4}\left(u_{i-1,j}+u_{i+1,j}+u_{i,j-1}+u_{i,j+1}\right)$

(c) $u_{ij}=\frac{1}{4}\left(u_{i-1,j-1}+u_{i+1,j+1}+u_{i+1,j-1}+u_{i-1,j+1}\right)$ (d) $u_{ij}=\frac{1}{4}\left(u_{i-1,j}+u_{i-1,j-1}+u_{i+1,j}+u_{i+1,j-1}\right)$

PART - B (5 x 2 = 10 Marks)

11. Find the Fourier constants b_n for x sinx in (-π , π).

12. If *F(f(x)) = F(s)* , then prove that $F\left[\frac{d^n(f(x))}{dx^n}\right]=(-is)^n F(s)$

13. State initial and final value theorems of Z transforms.

14. Classify $4u_{xx} + 4u_{xy} + u_{yy} - 6u_x - 8u_y - 16u = 0$

15. Derive the explicit difference equation corresponding to the partial differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Find the Fourier series for $f(x) = 1 + x + x^2$ in $(-\pi, \pi)$. Deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\infty = \frac{\pi^2}{6}$ (12)

(ii) Find the half range sine series for $f(x) = x$ in $(0, \pi)$. (4)

Or

(b) (i) Express $f(x) = x$ in half range cosine series and sine series in the range $0 < x < \ell$ and deduce the value of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots$ and $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots$ (8)

(ii) The values of x and the corresponding values of $f(x)$ over a period T are given below: Find a Fourier series upto first harmonic. (8)

x	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
$f(x)$	1.98	1.3	1.05	1.3	-0.88	-0.25	1.98

17. (a) (i) Find Fourier transform of $e^{-a^2x^2}$, $a>0$ and hence show that $e^{-x^2/2}$ is self-reciprocal. (8)

(ii) State and prove convolution theorem. (8)

Or

(b) (i) Find Fourier cosine transform of $f(x) = \begin{cases} \cos x & in \quad 0 < x < a \\ 0 & in \quad x \geq a \end{cases}$ and Fourier sine transform of $\frac{1}{x}$. (8)

(ii) Solve for $f(x)$ from the integral equation,

$$\int_0^\infty f(x) \sin sx \, dx = \begin{cases} 1 & for \quad 0 \leq s < 1 \\ 2 & for \quad 1 \leq s < 2 \\ 0 & for \quad s \geq 2 \end{cases}$$ (8)

18. (a) (i) Find $Z\,(t^2 e^{-t})$ and $Z\,(\sin^3 \frac{n\pi}{6})$ (8)

(ii) Find $Z^{-1}\left(\frac{z^2-3z}{(z-5)(z+2)}\right)$ using residue theorem. (8)

Or

(b) (i) Find $Z^{-1}\left(\frac{z(z^2-z+2}{(z+1)(z-1)^2}\right)$ by the method of partial fractions. (8)

(ii) Solve : $y_{n+2} - 3\,y_{n+1} + 2\,y_n = 2^n$ given that $y_o = y_1 = 0$. (8)

19. (a) A string is stretched between two fixed points at a distance 2ℓ apart and the points of the string are given initial velocities v where,

$$V = \begin{cases} \frac{cx}{l} \; in \; 0<x<l \\ \frac{c(2l-x)}{l} \; in \; l<x<2l \end{cases}$$

, x being the distance from an end point. Find the displacement of the string at any subsequent time. (16)

Or

(b) A bar 10 *cm* long with insulated sides, has its ends *A* and *B* kept at 50^o *C* and 100^o *C* respectively, until steady state conditions prevail. The temperature at *A* is then suddenly raised to 90^o *C* and at the same instant that at *B* is lowered to 60^o *C* and the end temperatures are maintained thereafter. Find the subsequent temperature function $u(x, t)$ at any time. (16)

20. (a) Solve $U_{xx} + U_{yy} = 0$, over the square mesh of side 4 units satisfying the following boundary conditions, by using Liebmann's iteration method by taking $h = k = 1$

(i) $U(0,y) = y^2/4$ for $0 \le y \le 4$

(ii) $U(4,y) = y^2$ for $0 \le y \le 4$

(iii) $U(x,0) = 0$ for $0 \le x \le 4$

(iv) $U(x,4) = 8 + 2x$ for $0 \le x \le 4$ (16)

Or

(b) (i) Solve $u_{xx} = 32\,u_t$ with h = 0.25 for $t > 0$; $0 < x < 1$ and $u(x,0) = u(0, t) = 0$;

$u(1, t) = t$. Tabulate u up to $t = 5$ sec using Bender-Schmidt formula. (8)

(ii) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $0 < x < 1$, $t > 0$, give $u(x, 0) = 0$, $u(0, t) = 0$, $u_t(x, 0) = 0$ and $u(1, t) = 100 \sin \pi t$ Compute u for 4 times steps with $h = 0.25$. (8)