Reg. No. : | | | | | | | | | | |

Question Paper Code: 41002

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2018

First Semester

Civil Engineering

14UMA102 - ENGINEERING MATHEMATICS – I

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 1 = 10 Marks)

1. If 1 and 2 are the eigen values of 2x2 matrix A. what are the eigen values of A^2.

   (a) 1 & 2 (b) 1 & 4 (c) 2 & 4 (d) 2 & 3

2. If two of the Eigen values of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ are 2 and 8, then the third Eigen value is

   (a) -2 (b) 0 (c) 2 (d) 3

3. Is this series convergent or divergent? $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+....$

   (a) convergent (b) divergent
   (c) Absolute converge (d) none of these

4. D’Alembert’s test is also called

   (a) Ratio test (b) Root test (c) Abel’s test (d) none of these

5. The radius of curvature of the curve $y = e^x$ at (0,1) is

(a) $2\sqrt{2}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{3}$

6. The envelope of the family of lines $y = x + a/m$, m being a positive integer

(a) $y^2 = 4ax$ (b) $x^2 = 4ay$ (c) $x^2 + y^2 = a^2$ (d) $xy = a^2$

7. If u = (x-y)(y-z)(z-x), then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} =$

(a) 0 (b) 1 (c) ∞ (d) none of these

8. If $x = r\cos\theta, y = r\sin\theta$, then $\frac{\partial(x,y)}{\partial(r,\theta)} =$

(a) 1 (b) r (c) r^2 (d) 0

9. $\int_0^a \int_0^b \int_0^c xyz\,dz\,dy\,dx$

(a) $\frac{a^2b^2c^2}{8}$ (b) $\frac{abc}{8}$ (c) abc (d) $a^2b^2c^2$

10. The value of the double integral $\int_0^\pi \int_0^a r\,dr\,d\theta$ is

(a) πa^2 (b) $\frac{\pi a^2}{2}$ (c) $\frac{\pi r^2}{2}$ (d) πr^2

PART - B (5 x 2 = 10 Marks)

11. Write down the quadratic form corresponding to the matrix $A = \begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$

12. Test for convergence of the series $\sum_1^\infty \left[\sqrt[3]{n^3+1} - n\right]$.

13. Find the radius of curvature at the point (c, c) on the curve xy = c^2.

14. If $x = u^2 - v^2$ and $y = 2uv$, find the Jacobian of x and y with respect to u and v.

15. Indicate the region of integration of $\int_{0}^{a}\int_{\frac{x^2}{a}}^{x} xdydx$.

PART - C (5 x 16 = 80 Marks)

16. (a) Verify Cayley Hamilton's theorem and hence find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (16)

Or

(b) Reduce the Q.F $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ in to a canonical form by an orthogonal transformation. (16)

17. (a) Discuss the convergence of the series whose n[th] term is $\frac{3.6.9.....3n}{4.7.10.......3n+1} \cdot \frac{2^n}{3n+2}$ (16)

Or

(b) Prove that if $b - 1 > a > 0$, the series $1 + \frac{a}{b} + \frac{a(a+1)}{b(b+1)} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)} + ...$ converges. (16)

18. (a) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta); y = a(1 - \cos\theta)$ is $4a\cos\frac{\theta}{2}$. (16)

Or

(b) Considering the evolute as the envelope of normals, find the evolute of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (16)

19. (a) Given the transformations $u = e^x \cos y \, and \, v = e^x \sin y$ and that ϕ is a function of u and v and also of x and y, prove that $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = (u^2 + v^2)\left[\frac{\partial^2\phi}{\partial u^2} + \frac{\partial^2\phi}{\partial v^2}\right]$. (16)

Or

(b) (i) Find the Taylor's series of $e^x \log(1 + y)$ in powers of x and y up to third degree terms. (8)

(ii) Find the maximum value of $f(x, y) = \sin x \sin y \sin(x + y); 0 \leq x, y < \pi$.. (8)

20. (a) Change the order of integration and hence evaluate it $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\,dy\,dx$. (16)

Or

(b) (i) By changing in to polar coordinates, evaluate $\int_0^a \int_y^a \frac{x}{x^2 + y^2}\,dx\,dy$. (8)

(ii) Find the volume of the tetrahedron bounded by the planes $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, $x = 0$, $y = 0$ and $z = 0$. (8)
