Reg. No. : | | | | | | | | | | |

Question Paper Code: 33021

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2018

Third Semester

Civil Engineering

01UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL Branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. Write down the Dirichlet's conditions for a function to be expanded as a Fourier series.

2. If the Fourier series of the function $f(x) = x + x^2$ in the interval $-\pi \le x \le \pi$ is $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{4}{n^2} \cos nx - \frac{2}{n} \sin nx \right]$, then find the value of the infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

3. Prove that if $F(s)$ is the Fourier transform of $f(x)$, then $F\{ f(x-a) \} = e^{isa} F(s)$.

4. Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$

5. Find Z transform of a^n.

6. State the convolution theorem of Z-transforms.

7. Write down the three possible solutions of one dimensional heat equation.

8. Classify the PDE $f_{xx} - 2f_{xy} = 0$.

9. Write the diagonal five point formula to solve the equation $u_{xx} + u_{yy} = 0$.

10. Write down the diagonal five point formula in Laplace equation?

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Find the Fourier series of *f(x)* = $\begin{cases} 1 & in \quad (0,\pi) \\ 2 & in \quad (\pi, 2\pi) \end{cases}$ Hence find the sum of the series $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} ..... \infty$ (8)

(ii) Find the half range cosine series of $f(x) = x sinx \; in (0,\pi)$. (8)

Or

(b) (i) Expand the function $f(x) = \sin x, 0 < x < \pi$ in a Fourier cosine series. (8)

(ii) Compute the first two harmonics of the Fourier series of *f(x)* given by (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	0.8	0.6	0.4	0.7	0.9	1.1	0.8

12. (a) (i) Find the Fourier transform of $f(x) = \begin{Bmatrix} 1 - |x| \; : \; |x| < 1 \\ 0 : otherwise \end{Bmatrix}$ and hence find the value of $\int_0^\infty \frac{sin^4 t}{t^4} \, dt$ (8)

(ii) Find the Fourier cosine transform of e^{-x^2} and hence find the Fourier sine transform of $x \, e^{-x^2}$. (8)

Or

(b) (i) Use Fourier transform technique to evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$ (8)

(ii) Find the Fourier sine transform of $f(x) = \begin{Bmatrix} x & : 0 < x < 1 \\ 2 - x & : 1 < x < 2 \\ 0 & : x > 2 \end{Bmatrix}$ (8)

13. (a) (i) Find $Z\left[\frac{2n+3}{n(n+1)}\right]$. (8)

(ii) Find the z – transform of $\frac{1}{(n+1)(n+2)}$ (8)

Or

(b) (i) Find $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ using convolution theorem. (8)

(ii) Solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 0$, $y_0 = -1$, $y_1 = 2$. (8)

14. (a) The ends A and B of a rod l cm long have the temperature at 30^0c and 80^0c until steady state prevails. The temperature of the ends is then changed to 40^0c and 60^0c respectively. Find the temperature distribution in the rod at any time. (16)

Or

(b) A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = 3(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at any time t. (16)

15. (a) Solve $u_{xx} = 32u_t$ for $t \geq 0$, $0 \leq x \leq 1$, $u(0, t) = 0$, $u(x, 0) = 0$ and $u(1, t) = t$ for two time step. (16)

Or

(b) Solve $u_{xx} + u_{yy} = 0$ over the square mesh of side 4 units; satisfying the following boundary conditions:

(i) $u(0, y) = 0 \; for \; 0 \leq y \leq 4$

(ii) $u(4, y) = 12 + y \; for \; 0 \leq y \leq 4$

(iii) $u(x, 0) = 3x \; for \; 0 \leq x \leq 4$

(iv) $u(x, 4) = x^2 \; for \; 0 \leq x \leq 4$ (16)

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