Reg. No. : | | | | | | | | | | |

Question Paper Code: 42002

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2018.

Second Semester

Civil Engineering

14UMA202 - ENGINEERING MATHEMATICS – II

(Common to ALL Branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 1 = 10 Marks)

1. The complete solution of $(D^3 - D)y = 0$ is

   (a) $y = A + Bx + Cx^2$ (b) $y = A + Bcosx + Csinx$

   (c) $y = A + Be^x + Ce^{-x}$ (d) $y = Ax + Be^{-x} + Ce^x$

2. The roots of $(D^2+2)y$ are

   (a) ± 2 (b) $\pm 2i$ (c) $\pm i\sqrt{2}$ (d) $\sqrt{2}$

3. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, the value of ∇r is

   (a) $\frac{\vec{r}}{r}$ (b) $\frac{r}{\vec{r}}$ (c) $\frac{1}{r}$ (d) $\frac{1}{\vec{r}}$

4. By stokes theorem, $\int_c \vec{r}\,\overrightarrow{dr} =$ _______

   (a) π (b) 1 (c) 0 (d) None of these

5. The derivative of f(z) at z_0 is

   (a) l (b) f(z) (c) $f(z_0)$ (d) $f'(z_0)$

6. The bilinear transformation that maps the points $\infty, i, 0$ onto $0, i, \infty$ is

(a) $-\frac{1}{z}$ (b) $-\frac{i}{z}$ (c) $\frac{i}{z}$ (d) None of these

7. Which of the following is not an analytic function?

(a) $\sin z$ (b) z (c) $\sinh z$ (d) $\bar{z}$

8. Conformal mapping is a mapping which preserves angle

(a) in magnitude (b) in sense

(c) both in magnitude and sense (d) Eithen in magnitude or in sense

9. $L^{-1}\left[\frac{1}{s^2+a^2}\right]=$

(a) $\frac{\sinh at}{a}$ (b) $\frac{\sin at}{a}$ (c) $\sinh at$ (d) $\sin at$

10. Laplace transforms is an ________ transform.

(a) Discrete (b) Discrete time

(c) Data independent (d) Integral

PART - B (5 x 2 = 10 Marks)

11. Solve $(D^4 - 2D^3 + D^2)y = 0$.

12. Prove that $\text{div}(\text{ curl }\vec{F}) = 0$.

13. Find the image of the circle $|z-2i| = 2$ under $w = \frac{1}{z}$.

14. State Cauchy's integral formula.

15. Find the Laplace Transform of $e^{-5t}t^2$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Solve the equation $(1+2x)^2y'' - 6(1+2x)y' + 16y = 8(1+2x)^2$. (8)

(ii) Solve the equation $(D^2+4D+3)y = e^{-x}sinx$. (8)

Or

(b) (i) Solve $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$. (8)

(ii) The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in 1 hour. What was the value of N after 3/2 hours? (8)

17. (a) Verify Stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper half surface of the sphere $(x^2 + y^2 + z^2) = 1$ and C is the circular boundary on Z = 0 plane. (16)

Or

(b) Verify the Gauss Divergence theorem for $\vec{F} = 4xz\,\vec{i} - y^2\vec{j} + yz\,\vec{k}$ over the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. (16)

18. (a) (i) Find the Bilinear transformation which maps the points $z = 0, -i, -1$ into $w = i, 1, 0$. (8)

(ii) If $f(z)$ is analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|\,f'(z)|^2$. (8)

Or

(b) (i) Prove that both real and imaginary part of an analytic function satisfy Laplace equation. (8)

(ii) Determine the analytic function $f(z) = u + iv$, if

$$u + v = \frac{sin2x}{cosh2y - cos2x}$$

(8)

19. (a) (i) Find the Laurent's series expansion of the function

$f(z) = \frac{7z-2}{z(z-2)(z+1)}$. In the annular region $1 < |z+1| < 3$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4sin\theta}$ using contour integration. (8)

Or

(b) (i) Using residue theorem, evaluate $\int_C \frac{3z^2+z-1}{(z^2-1)(z-3)}dz$ around the circle $x^2 + y^2 = 4$. (8)

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ using contour integration. (8)

20. (a) (i) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} sin\omega t & 0 < t < \frac{\pi}{\omega} \\ 0 & \pi < t < 2\frac{\pi}{\omega} \end{cases} \text{ with } f(t + 2\frac{\pi}{\omega}) = f(t).$$ (8)

(ii) Solve the differential equation $y'' + 2y' - 3y = \sin t$ given that $y(0) = y'(0) = 0$. (8)

Or

(b) (i) Solve $y'' + 4y' + 4y = e^{-t}$, y(0)= 0 and $y'(0) = 0$ using Laplace transform. (8)

(ii) Compute $y(1, 1)$ by using Runge-Kutta method of fourth order, given $\frac{dy}{dx} = y^2 + xy, y(1) = 1$. (8)