A **Reg. No. :** | | | | | | | | | | |

Question Paper Code: 51002

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2018

First Semester

Civil Engineering

15UMA102- ENGINEERING MATHEMATICS-I

(Common to ALL branches)

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\frac{d}{dx}(\sec h^{-1} x)$ is CO1- R

(a) $-\frac{1}{x\sqrt{1-x^2}}$ (b) $\frac{1}{x\sqrt{1-x^2}}$ (c) $-\frac{1}{x\sqrt{1+x^2}}$ (d) $\frac{1}{x\sqrt{1+x^2}}$

2. $\lim_{x \to 0} \frac{e^{ax}-e^{-ax}}{log\ (1+bx)}$ is CO1- R

(a) a/b (b) $a/2b$ (c) $2a/b$ (d) $2/b$

3. If $u = xyf\left(\frac{y}{x}\right) + yzf\left(\frac{y}{z}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ CO2- R

(a) u^2 (b) $3u$ (c) u (d) 2u

4. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$ CO2- R

(a)3 (b)6 (c)4 (d)-6

5. Value of $\Gamma\left(\frac{1}{2}\right)$ is CO3- R

(a) $\frac{\pi}{2}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) $\sqrt{\pi}$ (d) π

6. $\beta(m,n)\Gamma(m,n) =$ CO3- R

(a) $\Gamma(m)$ (b) $\Gamma(m)\Gamma(n)$ (c) $\beta(n)$ (d) $\beta(m)\beta(n)$

7. $\int_0^1 \int_0^2 \int_0^3 dzdydx$ is CO4- R

(a) 0 (b) 1 (c) 2 (d) 6

8. While changing Cartesian coordinates to polar coordinates in double integration, $dxdy$ is changed into CO4- R

(a) $drd\theta$ (b) $rdrd\theta$ (c) $\sin\theta\, drd\theta$ (d) $r\sin\theta\, drd\theta$

9. If $\begin{pmatrix}1\\2\\2\end{pmatrix}$ is an eigen vector of the matrix $\begin{pmatrix}4 & -2 & 1\\2 & 0 & 1\\2 & -2 & 3\end{pmatrix}$, then the corresponding eigen value is CO5- R

(a) -2 (b) -1 (c) 1 (d) 2

10. The Eigen values of, A^{-1} for CO5- R

$A = \begin{pmatrix}3 & 1\\0 & 2\end{pmatrix}$ is

(a) $\frac{1}{2}, 1$ (b) $\frac{3}{2}, \frac{1}{2}$ (c) $\frac{1}{3}, \frac{1}{2}$ (d) 3, 2

PART – B (5 x 2= 10 Marks)

11. Find

$\frac{dy}{dx}$ given $x = acos^3\theta;\ y = asin^3\theta.$ CO1- App

12. Write down the Taylor's formula to $f(x,y)$ in powers of x and y. CO2 -U

13. Evaluate

$\int_0^{\pi/2} sin^6\, xdx.$ CO3- E

14. Evaluate

$\int_0^a \int_y^a \frac{xdxdy}{x^2+y^2}$ by change of order. CO4 -E

15. Find the nature of the quadratic form $2x^2 + 4xy + 2y^2$. CO5- U

PART – C (5 x 16= 80Marks)

16. (a) If y= $e^{a\sin^{-1}x}$, *prove that* CO1- App (16)

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$

Or

(b) (i) Using Mcclaurin's series, expand $tanx$ by up to fifth degree. CO1 -App (8)

(ii) A corpse was found by a detective at exactly 8 P.M. The detective also measured the body temperature and found it to be 70F. Two hours later, the detective measured the body temperature again and found it to be 60F. If the room temperature is 50F, and assuming that the body temperature of the person before death was 98.6 F, at what time did the murder occur? CO1 -App (8)

17. (a) (i) Examine $f(x,y) = x^3 + y^3 - 12x - 3y + 20$ its extreme values. CO2 -App (8)

(ii) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2x_3}{x_1}, y_2 = \frac{x_3x_1}{x_2}, y_3 = \frac{x_1x_2}{x_3}$ CO2 -App (8)

Or

(b) Given the transformation $u = e^x \cos y$ *and* $v = e^x \sin y$ and that f is a function of u and v and also of x and y, prove that CO2- Ana (16)

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2+v^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right)$$

18. (a) (i) Evaluate $\int_0^{\pi} \frac{dx}{3+2sinx+cosx}$. CO3 -Ana (8)

(ii) Evaluate $\int_0^{\pi} \theta sin^3\theta \, d\theta$. CO3- Ana (8)

Or

(b) Prove that CO3- Ana (16)

$$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

19. (a) Change the order of integration and then evaluate CO4- Ana (16)

$\int_0^1 \int_{x^2}^{2-x} xydydx$.

Or

(b) (i) Find the area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. CO4- Ana (8)

(ii) Evaluate $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{3/2}}\, dxdy$. CO4- E (8)

20. (a) (i) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$. CO-5 U (8)

(ii) Reduce the quadratic form $2x^2 + y^2 + z^2 + 2xy - 2xz - 4yz$ into its canonical form. Hence find the rank, nature, index and signature. CO5- U (8)

Or

(b) Reduce the Quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to the canonical form through an orthogonal transformation and hence show that it is positive semi definite. Give also a non-zero set of values which will make the quadratic form zero. CO5- U (16)