Reg. No. : | | | | | | | | | | |

Question Paper Code: 41102

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2015.

First Semester

Civil Engineering

14UMA102 - ENGINEERING MATHEMATICS – I

(Common to all branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 1 = 10 Marks)

1. If $A = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 3 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ then eigen values of A^{-1} are

   (a) 2, 3, 5 (b) 2, 1, 4 (c) $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$ (d) $\frac{1}{2}, 1, \frac{1}{4}$

2. If 0, 3, 4 are eigen values of a square matrix A of order 3 then $|A| =$

   (a) 12 (b) 0 (c) ∞ (d) $\frac{1}{12}$

3. The harmonic series $\sum \frac{1}{n^p}$ is convergent if

   (a) $p > 1$ (b) $p < 1$ (c) $p = 1$ (d) $p \le 1$

4. $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n =$

   (a) n (b) 1 (c) e (d) $\frac{1}{e}$

5. The curvature of the curve $(x-2)^2+(y-3)^2=16$ at any point is

(a) 4 (b) $\frac{1}{4}$ (c) 6 (d) $\frac{1}{6}$

6. The envelope of the family of straight lines $at^2=ty-x, t$ is the parameter, is

(a) $y^2=4ax$ (b) $y^2=ax$ (c) $x^2=4ay$ (d) $x^2=ay$

7. Let u and v be functions of x, y and $u=e^v$. Then u and v are

(a) Functionally dependent (b) Functionally independent
(c) Functionally linear (d) Functionally non-linear

8. A stationary point of $f(x,y)$ at which $f(x,y)$ has neither a maximum nor a minimum is called

(a) Extreme point (b) Max-Min point
(c) Saddle point (d) Nothing can be said

9. The value of $\int_0^a \int_0^{\sqrt{a^2-x^2}} dy\,dx$ is

(a) $\frac{\pi a^2}{4}$ (b) $\frac{\pi a^2}{2}$ (c) $\frac{\pi a^2}{8}$ (d) πa^2

10. $\int_1^a \int_1^b \frac{dx\,dy}{x+y}=$

(a) $\int_1^b \int_1^a \frac{dx\,dy}{x+y}$ (b) $\int_1^b \int_1^a \frac{dy\,dx}{x+y}$ (c) $\int_1^b \int_0^a \frac{dy\,dx}{x+y}$ (d) $\int_1^1 \int_a^b \frac{dy\,dx}{x+y}$

PART - B (5 x 2 = 10 Marks)

11. Determine the nature of the quadratic form $x_1^2+3x_2^2+6x_3^2+2x_1x_2+2x_2x_3+4x_3x_1$ without reducing to canonical form.

12. State Leibnitz's test.

13. Find the radius of curvature of the curve $y=e^x$ at $x = 0$.

14. If u = $f(x-y, y-z, z-x)$, then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.

15. Evaluate $\int_1^2 \int_0^{x^2} x\,dy\,dx$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Find the eigen values and eigen vectors of the matrix A = $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$. (8)

(ii) Using Cayley Hamilton theorem,find the inverse of the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to the canonical form through an orthogonal transformation and hence show that it is positive semi-definite. (16)

17. (a) (i) Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.6} + \ldots$ (8)

(ii) Examine the convergence of the series $\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \ldots \quad (x > 0)$ (8)

Or

(b) Show that the series $\sum_{h=1}^{\alpha} \frac{(-1)^{n-1}}{2n-1}$ is conditionally convergent. (16)

18. (a) (i) Find the centre and circle of curvature for the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$. (8)

(ii) Find the envelope of the family $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are parameters connected by $a^2 + b^2 = c^2$, c is a constant. (8)

Or

(b) (i) Find the radius of curvature for the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. (8)

(ii) Show that evolute of the cycloid $x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$ is another cycloid. (8)

19. (a) (i) Find the Taylor's series expansion of $e^x \log(1+y)$ in powers of x, y upto the third degree terms. (8)

(ii) Find the volume of the greatest rectangular parallelepiped inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (8)

Or

(b) (i) If $g(x, y) = \psi(u, v)$, where $u = x^2 - y^2$, $v = 2xy$, then prove that

$$\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2}=4(x^2+y^2)\left(\frac{\partial^2 \psi}{\partial u^2}+\frac{\partial^2 \psi}{\partial v^2}\right).$$ (8)

(ii) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 where $y_1=\frac{x_2 x_3}{x_1}$, $y_2=\frac{x_1 x_3}{x_2}$, $y_3=\frac{x_1 x_2}{x_3}$. (8)

20. (a) (i) Change the order of integration in $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dx\,dy$ and hence evaluate it. (8)

(ii) Find the area lying between the parabola $y = x^2$ and the line $y = x$. (8)

Or

(b) (i) By changing in to polar coordinates, evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$. (8)

(ii) Find the volume of the tetrahedron bounded by the planes $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, $x = 0$, $y = 0$ and $z = 0$. (8)

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