Reg. No. : | | | | | | | | | | |

Question Paper Code: 41004

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2015.

Fourth Semester

Electronics and Communication Engineering

01UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

(Statistical tables may be permitted)

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Two dice are thrown together. Find the probability that (a) the total of the numbers on the top face is 9 and (b) the top faces are same.
2. Define exponential distribution.
3. If X and Y are two random variables with variance 2 and 3. Find the variance of $3X+4Y$.
4. State the equations of the two regression lines. What is the angle between them?
5. Prove that a first order stationary random process has a constant mean.
6. Consider a Markov chain with state {0, 1} transition probability matrix $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Is the state 0 periodic? If so, what is the period?
7. State any two properties of an auto correlation function.
8. Write down the Wiener-Khintchine theorem.
9. Describe a linear system with a random input.
10. Define White noise.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) The contents of bags I, II, III with balls are as follows. 1 white, 2 black and 3 red; 2white, 1 black and 1 red; 4 white, 5 black and 3 red. One bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from bags I, II and III? (8)

(ii) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distribution. (8)

Or

(b) (i) The density function of a random variable X is given by $f(x) = Kx\,(2 - x),\ 0 \le x \le 2$. Find K, mean, variance and rth moment. (8)

(ii) Define binomial distribution. Also obtain its moment generating function and hence the mean and variance. (8)

12. (a) (i) The joint probability density function of a random variable is given by

$$f(x, y) = \begin{cases} Kxye^{-(x^2+y^2)} & , x > 0, y > 0 \\ 0 \text{, otherwise} \end{cases}$$

. Find the value of K and prove also that x and y are independent. (8)

(ii) If X and Y are independent random variables with pdf $e^{-x}, x \ge 0$; $e^{-y}, y \ge 0$ respectively. Find the density function of $U = \dfrac{X}{X+Y}$ and $V = X + Y$. (8)

Or

(b) (i) Calculate the correction coefficient for the following dats. (8)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(ii) In a correction analysis, the equation of the two regression lines are $3x+12y=19$ and $3x+9y=46$. Find the value of the correction coefficient. (8)

13. (a) (i) Discuss the stationarity of the random process $X(t)=A\cos(\omega_0 t+\theta)$if A and ω_0 are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) The transition probability matrix of a markov chain with 3 states 0, 1, 2 is

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$ and the initial state distribution of the chain is

$P(X_0 = i) = 1/3,\ i = 0,1,2$.

Find (i) $P(X_2 = 2)$ and (ii) $p(X_3 = 1;\ X_2 = 2;\ X_1 = 1;\ X_0 = 2)$. (8)

Or

(b) (i) If the WSS process $\{X(t)\}$ is given by $X(t) = 10 \cos(100t+\Theta)$ where Θ is uniformly distributed over $(-\pi,\pi)$. Prove that $\{X(t)\}$ is correlation ergodic. (8)

(ii) Find the mean and autocorrelation of the Poisson process. (8)

14. (a) (i) If $\{X(t)\}$ is a WSS process with autocorrelation function,

$R_{xx}(\tau)$ and if $Y(t) = X(t + a) - X(t - a)$, show that

$R_{yy}(\tau) = 2 R_{xx}(\tau) - R_{xx}(\tau + 2a) - R_{xx}(\tau - 2a)$. (6)

(ii) Prove that $R_{xx}(\tau)$ is an even function of τ. (2)

(iii) The cross - power spectrum of real random processes $X(t)$ and $Y(t)$ is given by

$S_{XY}(w) = \begin{cases} a + jbw, & |w| < 1 \\ 0, & elsewhere \end{cases}$. Find the cross - correlation function. (8)

Or

(b) (i) The autocorrelation function for a stationary process $X(t)$ is given by $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean of random variable $Y = \int_0^2 X(t)dt$ and variance of $X(t)$. (8)

(ii) Consider two random process $X(t) = 3\cos(wt+\Phi)$ and $Y(t) = 2\cos(wt+\Phi-\pi/2)$ where Φ is a random variable uniformly distributed in $(0,2\pi)$. Prove that $\sqrt{R_{XX}(0)R_{YY}(0)} \geq |R_{XY}(\tau)|$. (8)

15. (a) (i) Given the power spectral density of the continuous process, $\dfrac{\omega^2 + 2}{\omega^4 + 13\omega^2 + 36}$.

Find the mean square value of the process. (8)

(ii) For a linear system with random input $x(t)$, the impulse response $h(t)$ and output $y(t)$, obtain the cross correlation function $R_{YX}(\tau)$ and the output autocorrelation function $R_{YY}(\tau)$. (8)

Or

(b) (i) Show that $S_{YY}(\omega) = |H(\omega)|^2 \, S_{XX}(\omega)$ where $S_{XX}(\omega)$ *and* $S_{YY}(\omega)$ are the power spectral density functions of the input *X(t)* and the output *Y(t)* and $H(\omega)$ is the system transfer function. (8)

(ii) The input to the RC filter is a white noise process with ACF $R_{XX}(\tau) = \frac{N_0}{2}\delta(\tau)$. If the frequency response $H(\omega) = \frac{1}{1 + j\omega RC}$, find the autocorrelation and the mean-square value of the output process Y(t). (8)
