Reg. No. : | | | | | | | | | | |

Question Paper Code: 31001

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2015.

Third Semester

Civil Engineering

01UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. State Parseval's theorem in Fourier series.
2. What do you mean by harmonic analysis in Fourier series?
3. Find the Fourier cosine transform of e^{-2x}.
4. State the relation between $F\{f(x)\}\ and\ F\{f(ax)\}$.
5. Define convolution of two sequences with respect to unilateral Z transforms.
6. Write the formula for $Z^{-1}[F(z)]$ using Cauchy's residue theorem.
7. What does a^2 represent in the equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$?
8. Write the appropriate solution of the one dimensional heat flow equation.
9. State the diagonal five point formula to solve the equation $u_{xx} + u_{yy} = 0$.
10. Write a difference formula for solving one dimensional wave equation $u_{tt} = a^2 u_{xx}$.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Find the Fourier series expansion of $f(x) = x^2 + x \ in \ (-2, 2)$. Hence find the sum of the series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \infty$ (8)

(ii) Find the Fourier series expansion of the function $f(x) = \begin{Bmatrix} 0 & : -\pi \leq x \leq 0 \\ sinx : 0 \leq x \leq \pi \end{Bmatrix}$ (8)

(b) (i) Find the half range cosine series of $f(x) = \begin{Bmatrix} x & : 0 < x < 1 \\ 2 - x : 1 < x < 2 \end{Bmatrix}$ (8)

(ii) Find the Fourier series of $y = f(x) \ in \ (0, 2\pi)$ up to third harmonic from the following values of $x \ and \ y$. (8)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

12. (a) (i) Find the Fourier transform of $f(x) = \begin{Bmatrix} 1 - |x| \ : \ |x| < 1 \\ 0 : otherwise \end{Bmatrix}$ and hence find the value of $\int_0^\infty \frac{sin^4 t}{t^4} \, dt$ (8)

(ii) Find the Fourier cosine transform of e^{-x^2} and hence find the Fourier sine transform of $x \, e^{-x^2}$. (8)

Or

(b) (i) Find the Fourier transform of $e^{-a|x|} \ if \ a > 0$ (8)

(ii) Find the Fourier sine transform of $f(x) = \begin{Bmatrix} x & : 0 < x < 1 \\ 2 - x : 1 < x < 2 \\ 0 \ : x > 2 \end{Bmatrix}$ (8)

13. (a) (i) Find the Z transform of $a^n \ cosn\pi \ \ and \ e^t \ sin2t$. (8)

(ii) Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given that$y_0 = y_1 = 0$, using Z transforms. (8)

Or

(b) (i) Find the inverse Z transform of $X(z) = \dfrac{2z}{(z-1)(z^2+1)}$ by Residue method. (8)

(ii) Find the inverse Z transform of $X(z) = \frac{8z^2}{(2z-1)(4z-1)}$ using convolution theorem. (8)

14. (a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = 3(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at any time t. (16)

Or

(b) A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite length. If the temperature at short edge $y = 0$ is given by $u = \begin{cases} 20\,x & : 0 \le x \le 5 \\ 20(10 - x) : & 5 \le x \le 10 \end{cases}$ and all the other three edges are kept at 0°C. Find the steady state temperature at any point of the plate. (16)

15. (a) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, y = 0,\ x = 3,\ y = 3\ with\ u = 0$ on the boundary and mesh length 1 unit. (16)

Or

(b) (i) Solve, by Crank-Nicholson method, the equation $u_{xx} = u_t$ subject to $u(x, 0) = 0, u(0, t) = 0\ and\ u(1, t) = t$ for two time steps by taking $h = 0.25$. (8)

`

(ii) Evaluate the pivotal values of the following equation taking $h = 1$ and up to one half of the period of the oscillation $16u_{xx} = u_{tt}$ given$u(0, t) = 0$, $u(5, t) = 0,\ u(x, 0) = x^2(5 - x)\ and\ u_t(x, 0) = 0$. (8)
