Reg. No. : | | | | | | | | | | |

Question Paper Code: 11002

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2015.

First Semester

Civil Engineering

01UMA102 - ENGINEERING MATHEMATICS – I

(Common to all branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Two of the Eigen values of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are 3 and 6. Find the Eigen value of A^{-1}.

2. State Cayley – Hamilton theorem and its uses.

3. Find the center and radius of the sphere $3(x^2+y^2+z^2)-2x-3y-4z-22=0$.

4. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 - x + 2y + z - 5 = 0$ at point (1, 1, 1).

5. Find the radius of curvature for $y = e^x$ at the point where it cuts the Y- axis (or) at x=0.

6. Find the envelope of the family of curve $y = mx + \frac{a}{m}$.

7. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

8. If $x = r\cos\theta$ and $y = r\sin\theta$, then find $\frac{\partial(r,\theta)}{\partial(x,y)}$.

9. Evaluate $\int_0^1\int_0^{x^2} (x^2+y^2)dydx$.

10. Evaluate $\int_0^1\int_0^2\int_0^3 xy^2z\ dzdydx$.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Find the Eigen values and Eigenvectors of the matrix A=$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$. (8)

(ii) Verify Cayley-Hamilton theorem find A^4 and A^{-1} when A=$\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $2x^2+y^2+z^2+2xy-2xz-4yz$ to canonical form by orthogonal reduction. Also find the nature of the quadratic form. (16)

12. (a) (i) Find the center, radius and area of the circle $x^2+y^2+z^2-2x-4y-6z-2=0$, $x+2y+2z=20$. (8)

(ii) Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$ $2x+3y+4z=8$; is a great circle. (8)

Or

(b) (i) Find the equation of the sphere passing through the circle $x^2+y^2+z^2+x-3y+2z-1=0$; $2x+5y-z+7=0$ and cuts orthogonally the sphere $x^2+y^2+z^2-3x+5y-7z-6=0$. (8)

(ii) Find the equation of the right circular cylinder whose axis is the line $x = 2y = -z$ and radius 4. (8)

13. (a) (i) Find the radius of curvature at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ on the curve $x^3+y^3=3axy$. (8)

(ii) Find the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point $\left(\frac{a}{4},\frac{a}{4}\right)$. (8)

Or

(b) (i) Find the equation of the evolute of the ellipse $\frac{x^2}{a^2}+\frac{x^2}{b^2}=1$. (8)

(ii) Find the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$, where a and b are connected by $a^2+b^2=c^2$, c being a constant. (8)

14. (a) (i) Given the transforms $u=e^x\cos y$ & $v=e^x\sin y$ and that ϕ is a function of u & v and also of x & y, prove that $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=(u^2+v^2)\left(\frac{\partial^2\phi}{\partial u^2}+\frac{\partial^2\phi}{\partial v^2}\right)$. (8)

(ii) Expand e^x cos y in powers of x and y as far as the terms of third degree using Taylor's expansion. (8)

Or

(b) (i) Examine $f(x,y)=x^3+y^3-12x-3y+20$ for its extreme values. (8)

(ii) A rectangular box open at the top is to have a volume of 32 c.c. Find the dimensions of the box, that requires the least material for its construction. (8)

15. (a) (i) Change the order of the integration and hence evaluate $\int_0^1\int_{x^2}^{2-x} xy\ dxdy$. (8)

(ii) Evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2+y^2)dydx$ by changing into polar coordinates. (8)

Or

(b) (i) Find, using a double integral, the area of the cardioids. $r=a(1+\cos\theta)$. (8)

(ii) Find the volume of that portion of the ellipsoid $\frac{x^2}{a^2}+\frac{x^2}{b^2}+\frac{z^2}{c^2}=1$, which lies in the first octant using triple integration. (8)
