Reg. No. [ ]

Question Paper Code : 27280

5 Year M.Sc. DEGREE EXAMINATION, MAY/JUNE 2016

Third Semester

Software Engineering

EMA 003 – PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS

(Common to 5 Year M.Sc. Software Systems)

(Regulations 2010)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Form the partial differential equation by eliminating a and b from

   $z = ax + a^2 y^2 + b + ab$.

2. Find $\frac{1}{D^2 - 2DD' + D'^2}(2e^{x+y})$

3. Find the sum of the Fourier series at $x = \pi$ for the function

   $$f(x) = \begin{cases} x - \pi, & 0 < x < \pi \\ -x, & \pi < x < 2\pi. \end{cases}$$

4. Find b_1 of the Fourier series of the function $f(x) = \begin{cases} x, & -1 \leq x \leq 0 \\ x + 2, & 0 \leq x \leq 1. \end{cases}$

5. If $F\left(e^{-\frac{x^2}{2}}\right) = e^{-\frac{s^2}{2}}$, find $F\left(xe^{-\frac{x^2}{2}}\right)$.

6. If $f\{f(x)\} = f(s)$, then show that $f\{f(x) \cos ax\} = \frac{1}{2}[f(s-a) + f(s+a)]$.

7. Find $L(1 - \cos^2 t)$.

8. Find $L^{-1}\left(\frac{b}{a}\left\{\frac{1}{b-s} + \frac{1}{a+s}\right\}\right)$.

9. Find $Z\left(\frac{e^{-at}}{n!}\right)$.

10. Find $Z^{-1}\left(\frac{3z}{z+2} - \frac{5z^2}{z^2+1}\right)$.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Form the partial differential equation by eliminating f and g from $z = f(y) + g(x + y + z)$. **(8)**

(ii) Solve $(D^2 + DD' + 6D'^2) z = \cos(3x + y)$. **(8)**

OR

(b) (i) Find the singular solution of $z = px + qy + \sqrt{1 + p^2 + q^2}$. **(8)**

(ii) Solve $(y - z) p + (z - x) q = x - y$. **(8)**

12. (a) Find the Fourier series of the function $f(x) = \begin{cases} 1 + \frac{x}{\pi} & \text{in } -\pi \leq x \leq 0 \\ 1 - \frac{x}{\pi} & \text{in } 0 \leq x \leq \pi \end{cases}$ and deduce the value of $\Sigma_0^\infty \frac{1}{(2n+1)^2}$. **(16)**

OR

(b) (i) Find the Fourier series of the function $f(x) = \begin{cases} -x & \text{in } -\pi \leq x \leq 0 \\ x & \text{in } 0 \leq x \leq \pi \end{cases}$ and deduce the value of $\Sigma_0^\infty \frac{1}{(2n+1)^4}$. **(8)**

(ii) Express $f(x) = x(\pi - x)$, $0 < x < \pi$, as a Fourier series of periodicity 2π containing sine terms only. **(8)**

13. (a) (i) Find the Fourier transform of $f(x) = e^{-ax}$, a > 0 and hence deduce that $\int_0^\infty \frac{\cos xt}{a^2+t^2}\, dt = \frac{\pi}{2} e^{-a|x|}$. **(8)**

(ii) Find the Fourier Cosine transform of $f(x) = \begin{cases} x, & 0 < x < 1, \\ 2-x, & 1 < x < 2, \\ 0, & x > 2 \end{cases}$. **(8)**

OR

(b) (i) Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$ and deduce that $\int_0^\infty \frac{\sin^4 t}{t^4}\, dt = \frac{\pi}{3}$. **(8)**

(ii) Using Parseval's identity, evaluate $\int_0^\infty \frac{x^2}{(x^2+4)(x^2+9)}\, dx$. **(8)**

14. (a) (i) Find $L\{t \cos^2 t\}$ and $L^{-1}\left\{\log \frac{s^2+a^2}{(s-b)^2}\right\}$. **(8)**

(ii) Using Convolution theorem, find $L^{-1}\left\{\frac{1}{s^3(s-a)}\right\}$. **(8)**

OR

(b) (i) Find the Laplace transform of $f(t) = \begin{cases} \sin wt, & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{\pi}{2w} \end{cases}$ and $f(t) = f\left(t + \frac{\pi}{2w}\right)$. **(8)**

(ii) Solve using Laplace transform method :

$(D^2 + D - 2)y = 20 \sin 2t$ given $y(0) = -1$, $y'(0) = 2$. **(8)**

15. (a) (i) Find $Z(n(n-1))$ and $Z(\cosh 3t \sin 2t)$. **(8)**

(ii) Solve by Z transform method : $y_{n+2} - 3y_{n+1} + 2y_n = 0$, $y_0 = 0$, $y_1 = 1$. **(8)**

OR

(b) (i) Find $Z^{-1}\left(\dfrac{z^2 + z}{(z-1)(z^2+1)}\right)$. **(8)**

(ii) Using Convolution theorem, find $Z^{-1}\left\{\dfrac{z^2}{(z-5)(z+2)}\right\}$. **(8)**