Reg. No. | | | | | | | | | | | | |

Question Paper Code : 27279

5 Year M.Sc. DEGREE EXAMINATION, MAY/JUNE 2016

Second Semester

Software Engineering

EMA 002 – ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Common to 5 Year M.Sc. Software systems)

(Regulations 2010)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Evaluate $\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta$.

2. Evaluate $\int_0^{1}\int_0^{2}\int_0^{3} xyz\,dx\,dy\,dz$.

3. Show that the vector $\vec{A} = (x^2 - y^2 + x)\,\vec{i} - (2xy + y)\,\vec{j}$ is irrotational.

4. Find a unit normal vector to the surface $x = t$; $y = t^2$; $z = t^3$ at $t = 1$.

5. Find the equation of the plane through (1, 2, 3) and parallel to the plane $4x + 5y - 3z + 7 = 0$.

6. Find the angle between the lines $\frac{x-1}{2} = \frac{y-3}{0} = \frac{z-1}{-1}$ and $\frac{x}{2} = \frac{y+1}{-1} = \frac{z+2}{1}$.

7. Prove that $u = \tan^{-1}(y/x)$ is harmonic.

8. Prove that the function $f(z) = e^{-z}$ is analytic.

9. Find the Taylor series expansion of $f(z) = \sin z$ about $z = \pi/4$.

10. Define essential singularity with an example.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Evaluate $\iint_R xy\,dx\,dy$, where R is the bounded by the parabola $y^2 = x$ and the lines $y = 0$ and $x + y = 2$, lying in the first quadrant. **(8)**

(ii) Evaluate $\iiint_V (xy + yz + zx)\,dx\,dy\,dz$, where V is the region of space bounded by $x = 0, x = 1, y = 0, y = 2, z = 0$ and $z = 3$. **(8)**

OR

(b) Change the order of integration in $\int_0^a \int_{x^2/a}^{2a-x} xy\,dy\,dx$ and then evaluate it. **(16)**

12. (a) (i) Find the directional derivative of $\phi = x^2yz + 4xz^2 + xyz$ at (1, 2, 3) in the direction of $2\vec{i} + \vec{j} - \vec{k}$. **(8)**

(ii) Prove that $\nabla^2 r^n = n(n+1) r^{n-2}$ and hence deduce $\nabla^2\left(\frac{1}{r}\right) = 0$. **(8)**

OR

(b) Verify divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b; 0 \le z \le c$. **(16)**

13. (a) Show that the liens $\frac{x-4}{5}=\frac{y-3}{-2}=\frac{z-2}{-6}$ and $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z-1}{-7}$ are coplaner. Find their point of intersection and the equation of the plane in which they lie. **(16)**

OR

(b) (i) Find the equation of the sphere passing through the points (1, 1, –2) and (–1, 1, 2) having its centre on the line $x + y - z - 1 = 0 = 2x - y + z - 2$. **(8)**

(ii) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$. Also find the point of contact. **(8)**

14. (a) (i) Find the analytic function f(z) = u + iv given $u = e^x [(x^2 - y^2) \cos y - 2xy \sin y]$; Also find V. **(10)**

(ii) An analytic function with constant modulus is also a constant. **(6)**

OR

(b) If f(z) = u + iv is analytic, find f(z) given that $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$. **(16)**

15. (a) (i) Evaluate $\int_C \frac{z+4}{z^2+2z+5}\, dz$, where C is the circle $|z + 1 + i| = 2$, using Cauchy's integral formula. **(8)**

(ii) Find the Laurent's series of $f(z) = \frac{z^2-1}{z^2+5z+6}$ valid in the region $2 < |z| < 3$. **(8)**

OR

(b) Evaluate $\int_0^\infty \frac{x^2\, dx}{(x^2+4)^2 (x^2+9)}$, using contour integration. **(16)**