

Question Paper Code: 27388

5 Year M.Sc. DEGREE EXAMINATION, MAY/JUNE 2016

Third Semester

Computer Technology

XCS 231 / 10677 SW 301 – PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS

(Common to 5 year M.Sc. Software Engineering and 5 Year M.Sc. Information Technology)

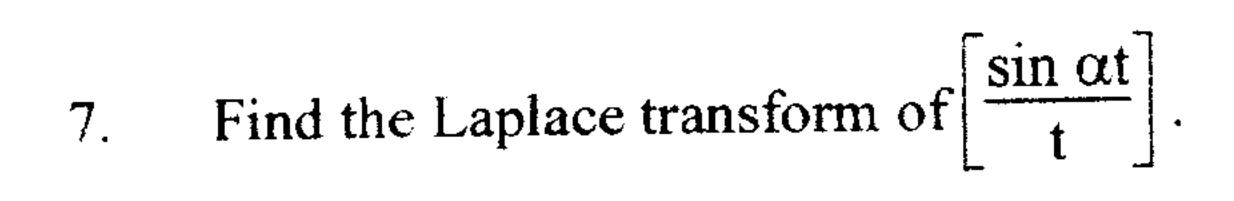
(Regulations 2003/2010)

Time: Three Hours

Maximum: 100 Marks

Answer ALL questions. $PART - A (10 \times 2 = 20 Marks)$

- 1. What is the partial differential equation in eliminating 'a' and 'b' from $(x-a)^2 + (y-b)^2 = z^2.$
- 2. Obtain the partial differential equation by eliminating f from $z = f(x^2 + y^2)$.
- 3. Find the Fourier constant b_n for $f(x) = x \sin x$ in $(-\pi, \pi)$
- 4. Find the RMS value of $y = x^2$ in $(-\pi, \pi)$.
- 5. Find the Fourier transform of $f(x) = e^{ikx}$, a < x < b.
- 6. Find the Fourier sine transform of $f(x) = e^{-x}$.



8. If
$$L[f(x)] = F(s)$$
 then prove that $L[xf(x)] = -\frac{d}{ds}[F(s)]$.

9. Prove that
$$Z\left[\frac{1}{(n+1)!}\right] = Z[e^{1/z} - 1].$$

10. Evaluate $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ using convolution theorem.

$$PART - B (5 \times 16 = 80 Marks)$$

11. (a) (i) Solve
$$x(y^2 + z) p - y(x^2 + z)q = (x^2 - y^2)z$$
.

(ii) Solve
$$(D^2 - DD' + D'^2)z = 2x + 3y$$
.

OR

(b) (i) Solve
$$(D^2 - 7DD' + 6D'^2)z = xy$$
.

(ii) Solve
$$(D^2 + 3DD' + 2D'^2)z = x + y$$
.

12. (a) (i) Form the partial differential equation by eliminating the arbitrary function
$$xyz = f(x + y + z)$$
.

(ii) Solve
$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x^2 y^2$$
.

OR

(b) (i) Solve
$$(mz - ny) p + (nx - lz)q = (ly - mx)$$

(ii) Solve
$$\frac{\partial^2 z}{\partial x^2} - 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = 2 \cosh (3x + y).$$

13. (a) (i) Find the inverse Fourier Transform of $\overline{f(s)}$ given by

$$\frac{1}{f(s)} = \begin{cases} a - |s|, & \text{for } |s| \le a \\ 0 & \text{for } |s| > a \end{cases}$$

Hence show that
$$\int_{0}^{\infty} \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}.$$
 (8)

(ii) Find the Fourier sine and cosine transforms of x^{n-1} . Hence deduce that $\frac{1}{\sqrt{x}}$ is self reciprocal under both the transforms. (8)

OR

(b) (i) Find the Fourier Transform of f(x), defined as

$$f(x) = \begin{cases} 1 - x^2 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases} \text{ Hence find } \int_{0}^{\infty} \left(\frac{\sin x - x \cos x}{x^3} \right) \cos \frac{x}{2} \, dx.$$
 (8)

(ii) Using transform method, evaluate
$$\int_{0}^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$$
. (8)

14. (a) (i) Find the inverse Laplace transform of $\frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)}$ (8)

(ii) Using convolution theorem, find the inverse Laplace transform of

$$\frac{1}{s(s^2+1)}$$
 (8)

OR

(b) (i) Solve:
$$f(t) = 4t - 3 \int_{0}^{t} f(u) \sin(t - u) du$$
. (8)

(ii) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} E, & \text{in } 0 \le t \le \frac{T}{2} \\ -E, & \text{in} \frac{T}{2} \le t \le T \end{cases}$$
 given that $f(t+T) = f(t)$. (8)

- 15. (a) (i) Find the z-transform of $\alpha^n \sin \alpha n$.
 - (ii) Use convolution theorem to find the inverse z-transform of

$$\frac{8z^2}{(2z-1)(4z+1)}$$

OR

- (b) (i) Find $z^{-1} \left(\frac{z^2}{(z+2)(z^2+4)} \right)$.
 - (ii) Find the initial and final values of f(n), if

$$\bar{f}(z) = \frac{0.4z^2}{(z-1)(z^2-0.736z+0.136)}$$