Reg. No. | | | | | | | | | | | | |

Question Paper Code : 27388

5 Year M.Sc. DEGREE EXAMINATION, MAY/JUNE 2016

Third Semester

Computer Technology

XCS 231 / 10677 SW 301 – PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS

(Common to 5 year M.Sc. Software Engineering and 5 Year M.Sc. Information Technology)

(Regulations 2003/2010)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. What is the partial differential equation in eliminating 'a' and 'b' from $(x-a)^2 + (y-b)^2 = z^2$.

2. Obtain the partial differential equation by eliminating f from $z = f(x^2 + y^2)$.

3. Find the Fourier constant b_n for $f(x) = x \sin x$ in $(-\pi, \pi)$

4. Find the RMS value of $y = x^2$ in $(-\pi, \pi)$.

5. Find the Fourier transform of $f(x) = e^{ikx}$, $a < x < b$.

6. Find the Fourier sine transform of $f(x) = e^{-x}$.

7. Find the Laplace transform of $\left[\frac{\sin \alpha t}{t}\right]$.

8. If $L[f(x)] = F(s)$ then prove that $L[xf(x)] = -\frac{d}{ds}[F(s)]$.

9. Prove that $Z\left[\frac{1}{(n+1)!}\right] = Z[e^{1/z} - 1]$.

10. Evaluate $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ using convolution theorem.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Solve $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$.

(ii) Solve $(D^2 - DD' + D'^2)z = 2x + 3y$.

OR

(b) (i) Solve $(D^2 - 7DD' + 6D'^2)z = xy$.

(ii) Solve $(D^2 + 3DD' + 2D'^2)z = x + y$.

12. (a) (i) Form the partial differential equation by eliminating the arbitrary function $xyz = f(x + y + z)$.

(ii) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x^2 y^2$.

OR

(b) (i) Solve $(mz - ny)p + (nx - lz)q = (ly - mx)$

(ii) Solve $\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x\,\partial y} + 2\frac{\partial^2 z}{\partial y^2} = 2\cosh(3x + y)$.

13. (a) (i) Find the inverse Fourier Transform of $\overline{f(s)}$ given by

$$\overline{f(s)} = \begin{cases} a - |s|, & \text{for } |s| \leq a \\ 0 & \text{for } |s| > a \end{cases}$$

Hence show that $\int_0^\infty \frac{\sin^2 x}{x^2}\, dx = \frac{\pi}{2}$. **(8)**

(ii) Find the Fourier sine and cosine transforms of x^{n-1}. Hence deduce that $\frac{1}{\sqrt{x}}$ is self reciprocal under both the transforms. **(8)**

OR

(b) (i) Find the Fourier Transform of f(x), defined as

$$f(x) = \begin{cases} 1 - x^2 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$. Hence find $\int_0^\infty \left(\frac{\sin x - x \cos x}{x^3}\right) \cos\frac{x}{2}\, dx$. **(8)**

(ii) Using transform method, evaluate $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$. **(8)**

14. (a) (i) Find the inverse Laplace transform of $\frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)}$. **(8)**

(ii) Using convolution theorem, find the inverse Laplace transform of $\frac{1}{s(s^2+1)}$. **(8)**

OR

(b) (i) Solve : $f(t) = 4t - 3\int_0^t f(u) \sin(t-u)du$. **(8)**

(ii) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} E, & \text{in } 0 \leq t \leq \frac{T}{2} \\ -E, & \text{in } \frac{T}{2} \leq t \leq T \end{cases}$$ given that $f(t+T) = f(t)$. **(8)**

15. (a) (i) Find the z-transform of $\alpha^n \sin \alpha n$.

(ii) Use convolution theorem to find the inverse z-transform of

$$\frac{8z^2}{(2z-1)(4z+1)}$$

OR

(b) (i) Find $z^{-1}\left(\frac{z^2}{(z+2)(z^2+4)}\right)$.

(ii) Find the initial and final values of f(n), if

$$\bar{f}(z) = \frac{0.4z^2}{(z-1)(z^2-0.736z+0.136)}$$
