Reg. No. [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 27383

5 Years M.Sc. DEGREE EXAMINATION, MAY/JUNE 2016

Second Semester

Computer Technology

XCS 122 – ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

[Common to 5 Year M.Sc. Software Engineering / 5 Year M.Sc. Information Technology]

(Regulations 2003)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Sketch the region corresponding to the integral $\int_0^1 \int_x^{x^2} f(x, y)\, dy\, dx$.

2. Evaluate : $\int_0^{\pi/2} \int_0^2 r\, dr\, d\theta$.

3. Give the physical meaning of Curl $\vec{F}$.

4. If $\bar{r}$ is the position vector of the point (x, y, z) and $f = x^2 + y^2 + z^2$, prove that If $\bar{r}$ grad $\phi = 2\phi$.

5. Find the equation of the plane which bisects at right angles the join of $(1, 3, -2)$ and $(3, 1, 6)$.

6. Find the equation of the sphere having the points (2, –3, 4) and (–1, 5, 7) as the ends of a diameter.

7. Prove that the function $f(z) = e^{-z}$ is analytic and also find their derivatives.

8. Prove that the function $u = e^{x} \cos y$ is harmonic.

9. State Cauchy's Residue theorem.

10. Find the poles of $f(z) = \dfrac{z^2}{(z^2 + 4)}$.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Evaluate $\iiint\limits_{V} \dfrac{dz\,dy\,dx}{\sqrt{1 - x^2 - y^2 - z^2}}$, where v is the region of space bounded by the co-ordinate planes and the sphere $x^2 + y^2 + z^2 = 1$ and contained in the positive octant. **(8)**

(ii) Evaluate $\int_{0}^{1} \int_{x}^{\sqrt{x}} xy\,(x + y)\,dx\,dy$. **(8)**

OR

(b) (i) Evaluate $\int_{0}^{1} \int_{x}^{1-x} \int_{x}^{1-x-y} xyz\,dx\,dy\,dz$. **(8)**

(ii) Change the order of integration in $\int_{0}^{1} \int_{y}^{2-y} xy\,dx\,dy$ and then evaluate it. **(8)**

12. (a) Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2dy)]$ where C is the boundary of the common area between $y = x^2$ and $y = x$. **(16)**

OR

(b) Verify Gauss divergence theorem for $\bar{F} = x^2\hat{i} + y^2\hat{j} + z^2k$ taken over the cube bounded by the planes by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. **(16)**

13. (a) (i) Find the length and equations of the shortest distance between the pair of lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. **(10)**

(ii) Find the equation of the plane passing through the point (1, 2, –1) and perpendicular to the planes $x + y - 2z = 5$ and $3x - y + 4z = 12$. **(6)**

OR

(b) (i) Find the equation of the sphere passing through the four points (1, 2, 3), (0, –2, 4), (4, –4, 2) and (3, 1, 4). **(10)**

(ii) Show that the lines $\frac{x-4}{2} = \frac{y-5}{3} = \frac{z-6}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. **(6)**

14. (a) (i) Prove that an analytic function whose imaginary part is constant is itself a constant. **(6)**

(ii) Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function. **(10)**

OR

(b) (i) Prove that $f(z) = \sin x \cdot \cosh y + i \cos x \cdot \sinh y$ is differentiable at every point. **(6)**

(ii) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x (\cos y - \sin y)$. **(10)**

15. (a) (i) Evaluate $\int_C \frac{z}{(z-1)(z-2)^2} dz$ where C is the circle $|z-2| = \frac{1}{2}$. **(8)**

(ii) Evaluate by contour integration $\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)} dx$. Where x is a real variable. **(8)**

OR

(b) (i) Find the Laurent expansion of the function $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < |z+1| < 3$. **(8)**

(ii) Evaluate by contour integration $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$. **(8)**