Reg. No. | | | | | | | | | | | | |

Question Paper Code : 27393

5 Years M.Sc. DEGREE EXAMINATION, MAY/JUNE 2016

Fourth Semester

Computer Technology

XCS 241/10677 SW 401 – DISCRETE MATHEMATICS

(Common to 5 Years M.Sc. Information Technology / 5 Years M.Sc. Software Engineering)

(Regulations 2003/2010)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Verify whether $(Q \wedge R) \rightarrow Q$ is a tautology.

2. What is meant by mathematical induction ?

3. Distinguish reflexive and irreflexive relations.

4. Define composition of two functions.

5. Define a group and give an example.

6. Find the left cosets of $\{[0], [3]\}$ in the group $<Z_6, +_6>$.

7. Define Field.

8. In an integral domain D, show that if $ab = ac$, with $a \neq 0$, then $b = c$.

9. State the distributive law in lattices.

10. Given an example of two elements Boolean algebra.

PART – B ($5 \times 16 = 80$ Marks)

11\. (a) (i) Show that the following premises are inconsistent. **(8)**

(1) If Jack misses many classes through fever, then he fails high school.

(2) If Jack fails high school, then he is uneducated.

(3) If Jack reads a lot of books, then he is not uneducated.

(4) Jack misses many classes through fever and he reads a lot of books.

(ii) Prove that $(\exists x)\,(P(X) \wedge Q(x)) \Rightarrow (\exists x)\, P(x) \wedge (\exists x)\, Q(x)$. **(8)**

OR

(b) (i) Obtain the principal disjunctive form and principal conjunctive form of the statement formula S given by

$P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$. **(8)**

(ii) Symbolize the following statements :

"If there was a ball game, then travelling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time. Therefore, there was no ball game". Show that these statements constitute a valid argument. **(8)**

12\. (a) (i) Explain matrix and digraph of a relation. **(4 + 4)**

(ii) Let I denote the set of all positive integeres and m be a positive integer. For x, y $\in$ I, define R = {<x, y> / x – y is divisible by m}. Show that R is an equivalence relation. **(8)**

OR

(b) (i) Prove that composition of functions is associative. **(4)**

(ii) If X = {1, 2, 3} and F_X denote the set of all bisections from X to X, find all the elements of F_X and find the inverse of each element. **(12)**

13\. (a) (i) If a and b are the elements of a group (G,*), then prove that

$(a * b)^{-1} = b^{-1} * a^{-1}$. **(8)**

(ii) Show that the (2, 5) encoding function defined by e(00) = 00000, e(01) = 01110, e(10) = 10101, e(11) = 11011 is a group code. **(8)**

OR

(b) (i) Prove that a subgroup H of a group G is normal if and only if $xHx^{-1} = H$ for all $x \in H$. **(8)**

(ii) Let G be a group and $a \in G$. Show that the map $f : G \to G$ defined by $f(x) = axa^{-1}$ for all $x \in G$ is a homomorphism. **(8)**

14. (a) (i) Show that every finite integral domain is a field.

(ii) Suppose f(t) is a polynomial over a field K and deg (f) = n. Then show that f(t) has atmost n roots.

OR

(b) (i) If p is a prime number, prove that Z_p is a field.

(ii) Let f(t) and g(t) be polynomials over a field K with $g(t) \neq 0$. Then prove that there exist polynomials g(t) and r(t) such that f(t) = q(t) g(t) + r(t), where either r(t) = 0 or deg (r(t)) < deg(g(t)).

15. (a) (i) Show that the operations of meet and join on a lattice are commutative, associative and idempotent. **(6)**

(ii) In any Boolean Algebra, show that

$(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$

$(a + b)(a' + c) = ac + a'b = ac + a'b + bc$ **(10)**

OR

(b) (i) Show that a lattice with three or fewer elements is a chain. **(6)**

(ii) Show that in a lattice if a <= b and c <= d, then a * c <= b * d. **(5)**

(iii) Show that in any Boolean Algebra, a = b if and only if ab' + a'b = 0 **(5)**