Reg. No. | | | | | | | | | | | | |

Question Paper Code : 27378

5 Year M.Sc. DEGREE EXAMINATION, MAY/JUNE 2016

First Semester

Computer Technology

XCS 112 – TRIGNOMETRY, ALGEBRA AND CALCULUS

(Common to 5 year M.Sc. Software Engineering/ 5 year M.Sc. Information Technology)

(Regulations 2003)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Show that $\left[\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right]^n = \cos n\theta + I\sin n\theta$.

2. If $z = e^{ia}$, find the value of $\frac{z^2-1}{z^2+1}$.

3. Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{pmatrix}$.

4. What are the applications of Cayley-Hamilton theorem ?

5. Find $\frac{du}{dx}$ if $u = \sin(x^2+y^2)$, where $x^2+4y^2=9$.

6. If $u = 2xy$, $v = x^2-y^2$, $x = r\cos\theta$ and $y = \sin\theta$, find $\frac{\partial(u,v)}{\partial(r,\theta)}$.

7. Evaluate $\int_0^{\pi/2} \sin^6 x \cdot \cos^4 x \, dx$.

8. Give the formula to find the surface area of the solid generated by the revolution of an arc of the curve $x = f(y)$ from $y = a$ to $y = b$ about y-axis. .

9. Find a differential equation of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ for which xe^x and e^x are the solutions.

10. Find the particular integral of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2^x$.

PART – B (5 × 16 = 80 marks)

11. (a) (i) Solve : $\frac{dx}{dt} + 2x - 3y = 5t, \frac{dy}{dt} - 3x + 2y = 2e^{2t}$.

(ii) Solve : $(x^2 D^2 - xD - 3)\, y = \sin(\log x)$.

OR

(b) (i) Solve the equation $\cos\left(\frac{\pi}{6} - x\right) = 0.88$. **(8)**

(ii) If $\cos(x + iy) = r(\cos\alpha + i\sin\alpha)$, prove that $y = \frac{1}{2}\log\frac{\sin(x-\alpha)}{\sin(x+\alpha)}$. **(8)**

12. (a) (i) Test for consistency and hence solve :

$x + y + z = 6, x + 2y - 2z + 3 = 0, 2x + 3y + z = 11.$

(ii) Find the Eigen values and the Eigen vectors of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

OR

(b) Reduce the following quadratic form into canonical form :

$3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1$.

13. (a) (i) Find the extreme values of $\sqrt{1 - x^2 - y^2}$. **(8)**

(ii) If $u = 4x^2 + 6xy$, $v = 2y^2 + xy$, $x = r\cos\theta$, $y = r\sin\theta$, evaluate $\frac{\partial(u, v)}{\partial(r, \theta)}$. **(8)**

OR

(b) (i) If $u = \tan^{-1}\frac{x^3 + y^3}{x - y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin u$. **(8)**

(ii) Expand $e^x \sin y$ in powers of x and y up to third degree terms. **(8)**

14. (a) (i) If $U_n = \int_0^{\pi/2} x^n \sin x \, dx$, $(n > 1)$, prove that U_n satisfies

$U_n + n(n-1)U_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$. Hence find U_5.

(ii) Find the area of the line segment cut-off from the parabola $x^2 = 8y$ by the line $x - 2y + 8 = 0$.

OR

(b) (i) Find the volume of the solid generated by the revolution of the cardioid $r = a(1 + \cos\theta)$ about the x axis.

(ii) Find the surface area of the solid generated by revolving the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ about the base.

15. (a) (i) Solve : $(D^2 + 2D - 1)\, y = (x + e^x)^2$, where $D \equiv \frac{d}{dx}$. **(8)**

(ii) Solve : $(x^3D^3 + 3x^2D^2 + xD + 1)\, y = \sin(\log x)$. **(8)**

OR

(b) Solve the simultaneous differential equations :

$Dx - (D - 2)\, y = \cos 2t$

$(D - 2)x + Dy = \sin 2t$, where $D \equiv \frac{d}{dt}$. **(16)**
