Reg. No. | | | | | | | | | | | |

Question Paper Code : 51769

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

First Semester

Civil Engineering

MA 2111/MA 12/080030001 – MATHEMATICS – I

(Common to all branches)

(Regulations 2008)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. The product of two eigen values of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value.

2. When is a Q.F. said to be singular ? What is its rank then ?

3. Find the equation of the sphere whose centre is (1, 2, –1) and which touches the plane $2x - y + z + 3 = 0$.

4. Find the radius of curvature of the curve $x^2 + y^2 - 4x + 2y - 8 = 0$.

5. Find the curvature of the circle $x^2 + y^2 = 25$ at the point (4, 3).

6. Define evolute of the curve.

7. If $u = \frac{x+y}{xy}$ find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

8. State Euler's theorem for homogeneous function.

9. Evaluate $\int_0^{\pi} \int_0^{\sin\theta} r\,dr\,d\theta.$

10. Change the order of integration in $\int_0^1 \int_0^{2\sqrt{x}} f(x, y)\, dy\, dx.$

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Obtain the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **(8)**

(ii) Using Cayley – Hamilton theorem, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$ and also verify the theorem. **(8)**

OR

(b) Reduce $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. Also find its rank, signature, index and nature. **(16)**

12. (a) (i) Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x - 2y + 6z + 5 = 0$ which are parallel to the plane $x + 4y + 8z = 0$. Find also their points of contact. **(8)**

(ii) Find the equation of the right circular cone whose vertex is (2, 1, 0), semivertical angle is 30° and the axis is the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z}{2}$. **(8)**

OR

(b) (i) Find the equation of the cylinder whose generators are parallel to $\frac{x}{2} = \frac{y}{2} = \frac{z}{-3}$ and whose guiding curve is the ellipse $3x^2 + y^3 = 3, z = 2$. **(8)**

(ii) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and also find the point of contact. **(8)**

13. (a) Find the equation of the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$. **(16)**

OR

(b) Find the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. **(16)**

14. (a) (i) If $u = \tan^{-1}\left[\frac{x^3 + y^3}{x - y}\right]$, using Euler's theorem on homogeneous functions, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. **(8)**

(ii) Find the maximum and minimum values of

$f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **(8)**

OR

(b) (i) Obtain the Taylor's series expansion of $x^3 + 4x^2y - 2xy^2 + y^3$ near the point $(-1, 1)$ upto the third degree terms. **(8)**

(ii) A rectangular box, open at the top, is to have a volume of 108 c.c. Find the dimensions of the box that requires the least material for its construction. **(8)**

15. (a) (i) Evaluate $\int\int xy\,dx\,dy$ over the region in the positive quadrant bounded by $\frac{x}{a} + \frac{y}{b} = 1.$ **(6)**

(ii) Find the value of $\int\int\int xyz\,dx\,dy\,dz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \leq a^2.$ **(10)**

OR

(b) (i) Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2 + y^2}\,dy\,dx$ and hence evaluate it. **(8)**

(ii) Evaluate, by changing to polar co-ordinates, the integral

$$\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} \frac{x^2 - y^2}{x^2 + y^2}\,dx\,dy.$$ **(8)**
