Reg. No. : | | | | | | | | | | |

Question Paper Code: 11002

B.E. / B.Tech. DEGREE EXAMINATION, OCTOBER 2014.

First Semester

Civil Engineering

01UMA102 - ENGINEERING MATHEMATICS – I

(Common to all branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. State Cayley - Hamilton theorem.

2. Find the Eigen values of A^{-1} if $A = \begin{bmatrix} 1 & 5 \\ 0 & 4 \end{bmatrix}$.

3. Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 6x + 8y - 8z - 1 = 0$.

4. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z - 6 = 0$ at (1,2,3).

5. Define curvature of a curve at a point.

6. Find the envelope of the family $y = mx + \frac{a}{m}$,where a is a constant.

7. If $x = u(1+v)$ and $y = v(1+u)$ find $\frac{\partial(x,y)}{\partial(u,v)}$.

8. If u= $e^{x}\sin y, where\ \ x = st^{2} and\ \ y = s^{2}t$, find $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$.

9. Evaluate: $\int_0^1 \int_0^2 xdydx$.

10. Evaluate: $\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} drd\theta$.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Find the Eigen values and Eigenvectors of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (8)

(ii) Verify Cayley – Hamilton theorem and hence find its inverse

$$A = \begin{bmatrix} 13 & -3 & 5 \\ 0 & 4 & 0 \\ -15 & 9 & -7 \end{bmatrix}$$ (8)

Or

(b) Reduce the following quadratic forms into the canonical form by an orthogonal transformation $6x^{2} + 3y^{2} + 3z^{2} - 4xy - 2yz + 4zx$ (16)

12. (a) (i) Find the equation of the sphere passing through the circle $x^{2} + y^{2} + z^{2} - 6x - 2z + 5 = 0,\ y = 0$ and touching the plane $3y + 4z + 5 = 0$. (8)

(ii) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. (8)

Or

(b) (i) Find the equation of the sphere having the circle $x^{2} + y^{2} + z^{2} + 10y - 4z - 8 = 0$ and $x + y + z = 3$ as a great circle. (8)

(ii) Find the equation of the cone whose vertex(1,2,3) and guiding curve is the circle $x^{2} + y^{2} + z^{2} = 4, x + y + z = 1$. (8)

13. (a) (i) Find the radius of curvature at $\left(\frac{3a}{2},\frac{3a}{2}\right)$ of the curve $x^3 + y^3 = 3$ axy. (8)

(ii) Find the equation of the circle of curvature of the parabola $xy = 12$ at (3, 4). (8)

Or

(b) (i) Show that the evolute of the cycloid $x = a\,(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another equal cycloid. (8)

(ii) Find the envelope of $\frac{x}{a}+\frac{y}{b}=1$, where the parameters a, b are related by $a^2 + b^2 = c^2$, c is a constant. (8)

14. (a) (i) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if

$$y_1 = \frac{x_2x_3}{x_1},\ y_2 = \frac{x_3x_1}{x_2},\ y_3 = \frac{x_1x_2}{x_3}.$$ (8)

(ii) In a triangle *ABC*, find the maximum value of *cosA ,cosB,and cosC.* (8)

Or

(b) (i) If $u = u(x, y)$ and $x = e^r \cos\theta, y = e^r \sin\theta$, Show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2r}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial \theta}\right)^2\right].$$ (8)

(ii) Expand e^x cosy in powers of x and y as far as the terms of the 3 rd degree. (8)

15. (a) (i) Change the order of integration and hence evaluate $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dydx$. (8)

(ii) By changing into polar coordinates find $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2)dy\,dx$. (8)

Or

(b) (i) Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = 4 - 4x$ as a double integral. (8)

(ii) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$ (8)

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